CS 1 -23〔Ｉ〕 A MICROMECHANICS-BASED COSTITUTIVE MODEL OF ICE AND FEM ANALYSIS FOR PREDICTION OF ICE FORCES.

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1.INTRODUCTION:. One of the challenging problem related to ice is the estimation of the maximum force that a moving ice sheet exerts on an off-shore structure. The solution requires information on the strength and deformation characteristics of polycrystalline ice under various condition of loading. Problems of ice are very complicated because ice exist at a temperature close to its melting point. The literature on ice-force prediction contains partly-empirical methods based on lower and upper bound theorems of plasticity which further requires the knowldge of the uniaxial compressive strength of ice, the reference stress method (power-law creep solutions) and theoretical models based on approximate analysis that idealize the ice sheet as a continuum plate on elastic foundation. Numerical simulations employing finite element method(FEM) also has been carried out with multiaxial power-law creep model. However, mode of failure and strength of ice were found to be governed by crack formation. Hence, information is required on the role that creep and crack formation play in the deformation and failure of ice and the dependence of these activity on stress, strain and time. In the present study a micromechanics-based constitutive model is developed to predict the time dependent behavior of ice and implemented in the FEM code to analyse an indentation problem.

2.PROPOSED MODEL: A sea-ice sheet is formed by columnar grain ice with randomly oriented C-axis in the horizontal plane. In practice, load acts perpendicular to the long axis of the columnar grains suggesting a two-dimensional analysis with plane stress condition. The elastic properties of a single crystal are different in different directions, however, overall texture of an ice sheet made up of randomly oriented crystal is isotropic. In the present model the stress-strain relation of the polycrystal ice is predicted on the basis of a single crystal property and cracking behavior is modelled based on the idea that local stress due to mismatch deformation of the grains causes micro cracking.

Consider the response of a single crystal under an applied load. Experimental results have shown that under applied stress, slip occurs along the basal planes. This slip depends on the resolved shear stess σ_{12} on the slip plane and is independent of the normal pressure on the plane. It is presumed here these non-reversible slip are the cause of the time dependent deformation of ice. The creep flow of single crystal ice is modelled by a power-law, $\epsilon^*_{12} = A\sigma^n_{12}t^m$. Since the basal plane is randomly oriented, the inelastic strain due to creep flow along the basal plane is inhomogeneous. The inhomogeneous inelastic strain induces local stresses resulting in the variation of the stress in each grain. The single grain is modelled as a homogeneous circular inclusion Ω embedded in an isotropic matix D, and by Eshelby's solutions the uniform strain inside the inclusion is given as $\epsilon_{ij} = S_{ijkl}(\epsilon^*_{kl} - \bar{\epsilon}^*_{kl})$ and associated uniform stress by Hook's law. The obtained stress is due to the difference between the inelastic strain inside an inclusion ϵ^*_{kl} and averaged inelastic stain of the matrix $\bar{\epsilon}^*_{kl}$. The total stress inside the inclusion is given by addition of applied stress. The creep behavior under arbitrary loading condition is described by adopting time-hardening law. Then the creep strain is given in the following rate form,

$$\dot{\epsilon}^*_{12} = Am[\sigma_{12} - \Theta(\epsilon^*_{12} - \bar{\epsilon}^*_{12})]^n t^{m-1}$$

where $\Theta = \frac{\mu}{2(1-2\nu)}$ represents the constraint for deformation of an individual crystal by the surrounding matrix. The macroscopic strain is calculated by taking average of strains in all grains with different basal plane orientations. The stress distribution just outside the inclusion is given by adding the stress jump derived by Goodier(1937), and the stress inside the inclusion. The maximum tensile stress criterion is adopted for crack nucleation and the crack density at time t is predicted. Once crack appears at the grain boundaries the constrain by the matrix is reduced and the grain deforms more freely. A simple damage model with the crack density ω (number of cracks over number of grain) as damage variable is adopted to degrade the virgin ice constrain Θ to $\bar{\Theta}$ linearly (i.e., $\bar{\Theta} = \Theta(1-\omega)$). This model prediction is justified by the comparision with experimental results.

3.RESULTS: The proposed consititutive model is used to predict the creep strain under constant applied stress as shown in the Fig.1. The declerating, steady and accelerating stages of the typical experimental observations are reflected in the model results. Figure 2 shows the predicted results for the constant strain rate conditions. This constitutive model is implemented in FEM code to analyze the practical problems. For example, the indentation test by Michel(1977) is simulated by the FEM mesh shown in Fig.3 and the results are compared in the Table.1.

4.CONCLUSIONS: A micromechanics-based constitutive model is formulated for polycrystalline ice. The model predictions are seen to fit uniaxial test results. The difference between the indentation test and model prediction is considered to be due to lack of information about the ice properties used in the test. Typical values for ice properties are used in the analysis.

5.REFERENCES:

[1]Michel,B and Toussaint,N;Mechanisms and Theory of Indentation of Ice Plates,Journal of Glacialogy,19(1977).

[2]Mura,T,Micromechanics of Defects in Solids, Martinus Nijhoff Publishers,1987.

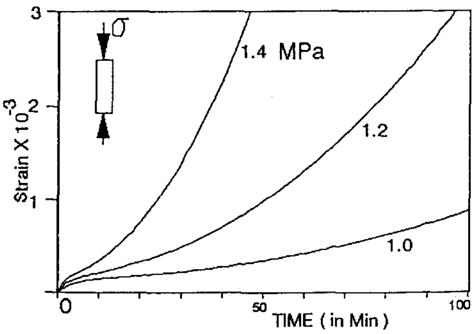

Fig.1 Theoretical creep curves for constant stress

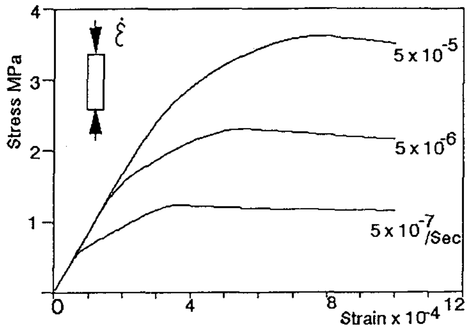

Fig.2 Theoretical stress-strain curves for constant strain rate

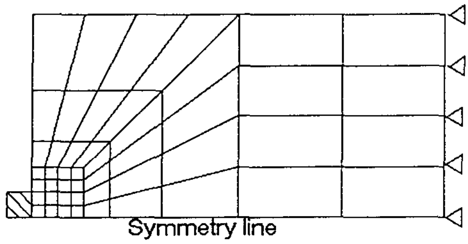

Fig.3 Mesh for finite element calculations

Indentation Rate (x b/sec)	Maximum strength (MPa)	
	Experimental	Theoretical
2.3×10^{-5}	4.62	6.06
2.0×10^{-4}	7.38	9.59
5.0×10^{-4}	9.83	12.08
1.9×10^{-3}	13.44	17.37

Table.1 Results of indentation tests and FEM (b=indentor width)