CS 1 -19〔I〕 A Continuum Model of Highly Jointed Rock Masses

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1. Introduction Rock masses are characterized by the existence of distributed joints. The mechanical properties of jointed rock masses are strongly dependent on the properties and geometry of joints. Constitutive modelling of jointed rock masses has been one of the subjects of interests to many investigators. There are two groups of models: discontinuum models and continuum models. Because of the tremendous number of joints in rock masses and the difficulty of geological survey of joints in situ, joint positions are known only statistically. Hence, it is impractical to model every joint in a deterministic way. Attention has been focused on development of equivalent continuum description, in which the jointed rock mass is treated as a continuum with effective material properties, while the effect of the joints is accounted for implicitly. Some fundamental problems related to the continuum modelling have to be solved in order to provide a general model for the numerical analysis of jointed rock masses. For example, the interaction between joints, the joint connection, and the complex joint mechanical behavior have to be considered. A continuum model of highly jointed rock masses is proposed which treats these problems.

2. A General Formulation of the Stress-Strain Relation An incremental constitutive relation is formulated by taking the volume average of stress and strain inside a *representative volume element* (RVE) as:

(1) $$\Delta\bar{\varepsilon}_{ij} = C^R_{ijkl}\Delta\bar{\sigma}_{kl} + \frac{1}{2V}\sum_k \int_{S^J_k} \left(\Delta[u_i]n_j + \Delta[u_j]n_i \right)_k dS ,$$

where C^R_{ijkl} is the compliance tensor of the matrix, S^J_k is the surface of joint k , n_i is the unit normal vector of the joint surface, and $\Delta[u_i]$ is the incremental displacement jump across the joint surface, which is related to the average incremental stress over the joint through a constitutive law of joints. An elasto-plastic joint constitutive law, which is based on the classical theory of plasticity, is used in the model. If the increment of traction on the joint is expressed in terms of the average stress in the RVE as:

(2) $$\{\Delta\bar{\sigma}^J\}'_k = \left[F_{ij}\right]'_k \{\Delta\bar{\sigma}\}' ,$$

with $\left[F_{ij}\right]'_k$ being called *joint stress concentration tensor* (SCT), then the complete incremental constitutive relation is obtained. The SCT depends on the ratio of the joint stiffness to the system stiffness which is the stiffness of the surroundings of the joint. A simple method is developed to evaluate the SCT.

3. Consideration of the Interaction Effect When evaluating the displacement jumps due to one joint, the presence and influence of other joints is considered as the interaction effect. The interaction effect cannot be neglected in the modelling of highly jointed rock masses because it has a great influence on the overall response of the jointed rock masses. Instead of using methods such as the self-consistent method or the differential scheme method, we adopt the homogenization method proposed by Cai and Horii (1991) to consider the interaction effect. Firstly, the tangential compliance of the jointed rock mass is obtained by using the non-interaction solution, which is simple and straightforward. Then each joint is embedded in the effective material with the tangential compliance tensor obtained from the non-interaction solution to evaluate the SCT, and subsequently the final overall tangential compliance tensor by summarizing the contribution from every joint. It has been demonstrated that the method used for considering the interaction effect is simple and the result is in good agreement with experimental data.

4. Consideration of Joint Connection Joints are often found to be connected to each other if there are more than two sets of joints exist in the rock mass. Joint connection usually results in more deformation of the rock mass. Up to now, there is no proper way to treat such kind of effect in the continuum modelling of jointed rock masses. The concept of *system stiffness* is introduced in order to estimate the SCT. The original problem of a joint in an RVE is decomposed into a homogeneous problem, a sub-problem of open slit with non-zero traction and the cut-out joint itself (see Fig.1). The sub-problem is further simplified with the introduction of the concept of system stiffness, which gives the relationship between the average displacement jump of an open slit and the average traction acting on it. Based on the dimensional consideration and the penny-shaped crack solution, the system stiffnesses for an isolated joint in an isotropic elastic intact matrix are of the form,

$$K^n = \frac{E}{\lambda_0^n L^J}, \quad K^s = \frac{G}{\lambda_0^s L^J}, \quad K^t = \frac{G}{\lambda_0^t L^J}, \tag{3}$$

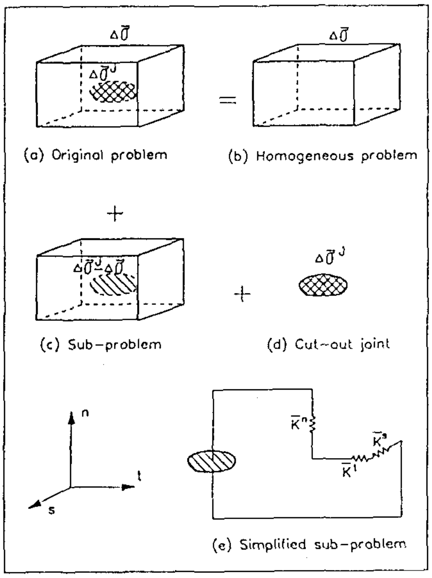

Fig.1 A simple model for estimation of SCT

where E and G are the Young and shear moduli of the intact rock, respectively; L^J is the characteristic length of the joint; λ_0^n, λ_0^s, λ_0^t are dimensionless factors depending on the shape of joints. The SCT is obtained from the displacement compatibility condition in the decomposition. When the joint is embedded in the jointed material, the stiffness of the system surrounding the joint is reduced and its property becomes anisotropic. Furthermore, when there are many joints inside the RVE, the possible joint connection will decrease the stiffness of the matrix or increase the effective joint length, resulting in the reduction of the system stiffness. So that an effective way that we proposed to treat the joint connection is to replace λ_0^n, λ_0^s, λ_0^t in Eqn.(3) by the connectivity coefficients λ^n, λ^s, λ^t. Then, the following system stiffness are assumed for a joint in the jointed rock mass as:

$$\bar{K}^n = \frac{\bar{E}}{\lambda^n L^J}\beta^n, \quad \bar{K}^s = \frac{\bar{G}^s}{\lambda^s L^J}\beta^s, \quad \bar{K}^t = \frac{\bar{G}^t}{\lambda^t L^J}\beta^t, \tag{4}$$

where $\bar{E}$ is the tangential effective Young's modulus in the direction normal to the joint and $\bar{G}^s$, $\bar{G}^t$ are the tangential effective shear moduli for the shear deformation in n-s plane and n-t plane, respectively. β^n, β^s and β^t are the modification coefficients for the anisotropic behavior of the effective matrix. The connectivity coefficients can be identified through the comparison of model prediction with the experimental data or some numerical simulation as is demonstrated in Cai and Horii (1992), Kubota and Horii (1992). This treatment of the joint connection effect is proved to be appropriate.

5. Conclusion Some of the fundamental problems related to the continuum modelling of highly jointed rock masses are discussed and possible ways of solving them are suggested. For the complicated jointed rock masses, the continuum model offers a powerful analytical procedure.

Reference

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