IV-67 Fuzzy PERT: Different Fuzziness for different activity

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1 Introduction

In FPERT [1] four types of activity fuzziness are introduced: type 1: the time estimate t_{ij} is determinant; type 2: the considered activity can be finished within D_{ij}; However, if it is expedited, it must be finished within d_{ij}; type 3: the considered activity can be finished within d_{ij}. However, if it is delayed, it must be finished within D_{ij}; type 4: it is desired to get the considered activity completed within the period between τ_{1ij} and τ_{2ij}. However, if the activity was expedited or delayed, the duration tij must not exceed d_{ij} or be less than D_{ij} respectively. In FPERT, it is assumed that existing fuzziness can be expressed by one value "λ". In fact, this value may not be the same for all activities regarding the particular characteristics of each one. In this paper, FPERT is developed to consider the different levels of fuzziness λ_{ij} do exist in real construction works.

2 LP Formulation

In FPERT with different fuzzy levels, based on judgment of the decision maker, many levels of fuzziness λ_{ij} are proposed. Two approaches are introduced in this research to determine λ_{ij}'s values. **The former** is to determine them outside the model. Namely, the user of the model will decide by his knowledge or experience what fuzzy levels should λ_{ij}'s values be assigned. These levels will be the new constrains of the model. **The later** is to determine them inside the model. That is, let the model decides them. For the example under consideration only the first approach is demonstrated and the following fuzzy levels are given : $\lambda_1 \geq 0.8$, $\lambda_2 \geq 0.7$, $\lambda_3 \geq 0.6$. They are the new constraints.

The new model of **FPERT Earliest Start Schedule** with different fuzzy levels can be defined as follows:

Maximize $\quad Z_0 = \sum_{(i,j)\in W} \lambda_{ij}$

Subject to $\quad S_{01} = 0$

$$F_{ik} \leq S_{kj} \quad (k \in N, i \in S_k, j \in P_k)$$

$$\lambda_{ij} \geq V_{ij}$$

Type 1: $\quad F_{ij} - S_{ij} = t_{ij}$

Type 2: $\quad 1 - \frac{D_{ij}-(F_{ij}-S_{ij})}{D_{ij}-d_{ij}} \geq \lambda_{ij}$

Type 3: $\quad 1 - \frac{(F_{ij}-S_{ij})-D_{ij}}{D_{ij}-d_{ij}} \geq \lambda_{ij}, \quad F_{ij} - S_{ij} \geq d_{ij}$

Type 4: $\quad 1 - \frac{\tau_{1ij}-t_{ij}}{\tau_{1ij}-d_{ij}} \geq \lambda_{ij}$ and $1 - \frac{t_{ij}-\tau_{2ij}}{D_{ij}-\tau_{2ij}} \geq \lambda_{ij}$

$$\sum_{((i,j)\in W)}(S_{ij} + F_{ij}) \leq Z$$

And $\quad F_{ij}, S_{ij} \geq 0, \ ((i,j) \in W), \quad \lambda_{ij} \geq 0.$

In Which: W network activities, N network nodes.
$i < j$ signifies node i proceeds node j, and the project starts at node 1 and ends at node n.
$i \in S_k$ nodes succeed node $k \in n$, and $j \in P_k$ nodes proceed it.

3 Example

For the network under consideration the different time estimates are shown in Table 1 and the different fuzzy levels are shown in Table 2. The simplex method is used to solve this linear programming problem when the new constraints are at their lowest levels. The result of calculations which gave $Z = 409.1$ and $F_{n,n+1} = D = 35.9$ is shown in Figure 1. Theoretically, for the network under consideration, Z can be increased from its lowest value 409.1 gradually, and the curve which shows the relationship with λ_{ij}'s values is drawn in Figure 2. In general, the previous curve can be used as a graphical method to get activities time estimates for a particular values of λ_{ij}. Instead, the trial and error as a mathematical way can be used too.

The new model of **FPERT Latest Completion Schedule** can be defined by imposing the network constraints of Latest Completion Schedule, that is, $F_{n,n+1} = D \ and \ \sum_{((i,j)\in W)}(S_{ij}+F_{ij}) \geq Z$. The same steps of calculations can be applied to solve this model. However, using the same value of Z gave different values of λ_{ij}. In addition, it violated the network logic with $S_{01} \neq 0$. Actually, the values of λ_{ij} do depend on the value of Z, and using the trial and error method, the same values of λ_{ij} and activities durations of Earliest Start Schedule could be obtained for the Latest Completion Schedule at $Z = 505.1$. In fact, due to difficulties in computational process, it might not be desirous to use this model to control λ_{ij}' values; both Z *and* D are involved in the trial and error calculations.

4 Conclusions

It is assumed that, based on the estimator's knowledge and experience, λ's value should reflect the level of fuzziness regarding many factors do affect the estimation process. λ_{ij}'s values can be used as a measure to select the best alternatives. Generally, high value of λ would express a high level of belief that the considered project or activity would be executed within the estimated duration.

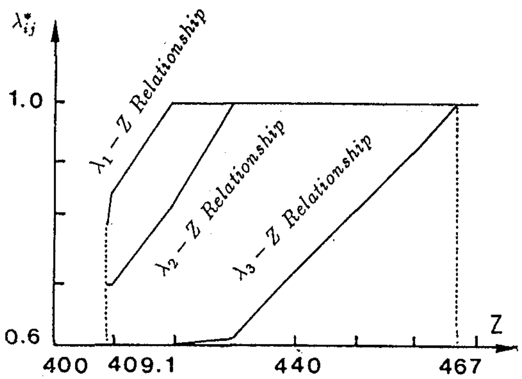

Figure 2: $\lambda^*_{ij} - Z$ FPERT ESS

Table 2: Time Estimate of Activities

Act.	Type	d_{ij}	τ_{1ij}	τ_{ij}	τ_{2ij}	D_{ij}
0,1	1			0		
1,2	2	3				6
1,3	3	5				10
2,4	4	3	5		8	10
3,4	1			4		
3,5	4	10	15		17	20
4,5	1			0		
4,6	3	7				15
4,7	2	15				25
5,6	2	6				9
5,7	1			5		
6,7	3	6				12
7,8	1			0		

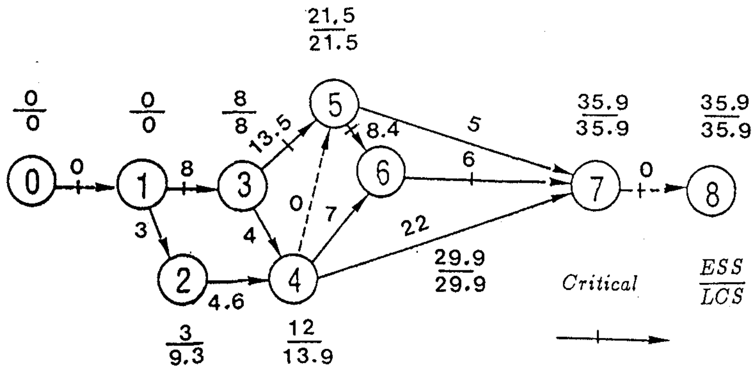

Table 1: λ's Values

Act.	λ	Act.	λ
0,1	Det.		
1,2	λ_1	4,6	λ_3
1,3	λ_3	4,7	λ_2
2,4	λ_1	5,6	λ_1
3,4	Det.	5,7	Det.
3,5	λ_2	6,7	λ_2
4,5	Det.	7,8	Det.

Figure 1: Critical Path Calculations of FPERT

5 References

[1] Chishaki, T., and Tatish, M. "FPERT" Technology Reports of Kyushu University. Vol. 63, No. 2, pp. 109-115, 1990.