II-465 MASS TRANSPORT UNDER WAVE-CURRENT COMBINDED MOTIONS

o Seree Supharatid, Graduate student,
H. Tanaka, Associate Professor,
N. Shuto, Professor, Tohoku Univ.

ABSTRACT

An analytical-numerical model of mass transport velocity in a turbulent boundary layer under wave-current combination is presented. The model is used to predict the Eulerian mass transport velocity obtained from the authers' experimental results.

I FORMULATION OF THE PROBLEM

The following boundary layer equation is used with the assumption that the free stream outside the boundary layer is composed of a steady uniform current(u_c) combined with an oscillatory flow(U_α).

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial \tau/\rho}{\partial z} \qquad (1)$$

When the horizontal velocity in the boundary layer is expanded, the first term u_c gives the primary steady component and the oscillatory component u_1 is of the order ε.

$$u = u_c + \varepsilon u_1 + \varepsilon^2 u_2 + \ldots\ldots\ldots\ldots \qquad (2)$$

The perturbed equations are given as follows, on assuming a linearly time-invariant eddy viscosity, i.e. $\tau/\rho = \kappa u_{*wc} z\, \partial u/\partial z$.

$$\varepsilon^0 : \qquad 0 = -\frac{1}{\rho}\frac{\partial p_c}{\partial x} + \frac{\partial}{\partial z}\kappa u_{*wc} z\frac{\partial u_c}{\partial z} \qquad (3)$$

$$\varepsilon^1 : \qquad \frac{\partial u_1}{\partial t} + u_c\frac{\partial u_1}{\partial x} - \frac{1}{\xi}\frac{\partial u_c}{\partial \xi}\int_{\xi_0}^{\xi}\xi\frac{\partial u_1}{\partial x}d\xi = \frac{\partial U_\alpha}{\partial t} + \frac{\omega}{\xi}\frac{\partial}{\partial \xi}\xi\frac{\partial u_1}{\partial \xi} \qquad (4)$$

$$\varepsilon^2 : \qquad \frac{\partial u_2}{\partial t} + u_c\frac{\partial u_2}{\partial x} + u_1\frac{\partial u_1}{\partial x} - \frac{1}{\xi}\frac{\partial u_1}{\partial \xi}\int_{\xi_0}^{\xi}\xi\frac{\partial u_1}{\partial x}d\xi - \frac{1}{\xi}\frac{\partial u_c}{\partial \xi}\int_{\xi_0}^{\xi}\xi\frac{\partial u_2}{\partial x}d\xi$$

$$= U_\alpha\frac{\partial U_\alpha}{\partial x} + \frac{\omega}{\xi}\frac{\partial}{\partial \xi}\xi\frac{\partial u_2}{\partial \xi} \qquad (5)$$

where $\xi = 2(\omega z/\kappa u_{*wc})^{1/2}$, u_{*wc} = maximum combined friction velocity

We further assume that u_c is much smaller than the wave celerity. Then, the second and third terms on the left-hand side of Eq. (4) can be neglected.

The oscillatory component in the free stream is expressed as $U_\alpha = \varepsilon U_0(x)e^{-i\omega t}$. Solutions of Eqs. (3) and (4) are obtained for the boundary conditions(u_c = 0 at z = zo, and $\partial u_c/\partial z = 0$ at z = D; u_1 = 0 at $\xi = \xi_0$ and $u_1 = U_\alpha$, $\xi \rightarrow \infty$) as follows.

$$u_c = \frac{u_{*c}^2}{\kappa u_{*wc}}\left\{\ln\frac{z}{zo} - \frac{(z - z_0)}{D}\right\} \qquad (6)$$

$$u_1 = U_0(1 - F)e^{-i\omega t}, \quad F = \frac{\mathrm{Ker}\xi + i\mathrm{Kei}\xi}{\mathrm{Ker}\xi + i\mathrm{Kei}\xi} \qquad (7)$$

If time-averaged, Eq. (5) is reduced to

$$\overline{u_1\frac{\partial u_1}{\partial x}} - \overline{\frac{1}{\xi}\frac{\partial u_1}{\partial \xi}\int\xi\frac{\partial u_1}{\partial x}d\xi} = \overline{U_\alpha\frac{\partial U_\alpha}{\partial x}} + \frac{\omega}{\xi}\frac{\partial}{\partial \xi}\xi\frac{\partial \overline{u_2}}{\partial \xi} \qquad (8)$$

Substitution of the real part of u_1 and some manipulations lead to

$$\frac{\omega}{\xi}\frac{d}{d\xi}\xi\frac{d\overline{u_2}}{d\xi} = U_0^* \frac{\partial U_0}{\partial x}\{|F|^2 - F - F^* + \frac{1}{2\xi}F''(\xi^2-\xi_0^2) + i|F'|^2\} - i\frac{\xi_0}{\xi}F''^* F'|_{\xi=\xi_0} \quad (9)$$

where F is the Kelvin function in Eq. (7). Supercripts * and ' mean the complex conjugate and the derivation with respect to ξ.

For a numerical solution, u_2 is expressed as $Re\{(1/\omega)G(\xi)U_0^*\partial U_0/\partial x\}$ and Eq. (9) is reduced to

$$\frac{dG}{d\xi} = -3Im\{F'\} + \xi\frac{F^*}{2} - i\frac{\xi_0}{\xi}F^*F'|_{\xi=\xi_0} \quad (10)$$

Equation (10) is numerically solved by the Runge-Kutta method, to satisfy the boundary condition(u_2 = 0, $\xi = \xi_0$)

II RESULTS

For a progressive wave $\eta = (H/2)e^{i(kx-\omega t)}$ on the water of constant depth, D, the free stream velocity is

$$U_\alpha = \frac{H\omega}{2\sinh(kD)}e^{i(kx-\omega t)} \quad (11)$$

and $\overline{u_2}$ is given by

$$\overline{u_2} = \frac{H^2k\omega}{4\sinh^2(kD)}Im\{G(\xi)\} \quad (12)$$

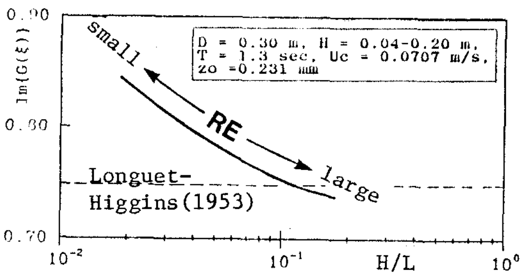

Fig. 1 Im{G(ξ)}

A comparison of the present theory and the theory of Longuet-Higgins(1953) is shown in Fig. 1, for the velocity at the outer edge of boundary layer. The present theory gives u_2 variable with ξ or with H/L which was observed in experiments by Bijker et al. (1974), while the L.H. theory gives a constant 3/4 independent of ξ. The computed results also suggest that the difference from the L.H. theory can be expressed in terms of a Reynolds number defined with the water particle amplituide at the bottom and U_0.

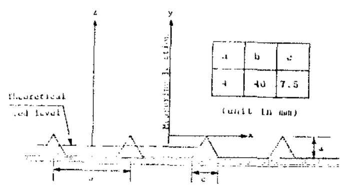

Fig. 2 Bed configulation

Hydraulic experiments were carried out on a rough bed shown in Fig. 2. Fig. 3 compares the vertical distribution of mass transport velocity. The present theory(PRE) seems to slightly improve in the very neighborhood of the bottom. None of the theories, however, can predict the negative velocity near the bottom, which might be generated under a strong effect of vortices ejected from crests of triangular roughnesses.

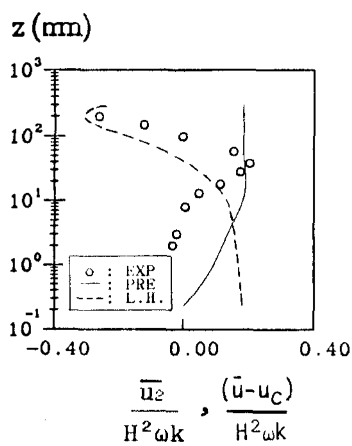

Fig. 3 Eulerian mass transport velocity

REFERENCE

Longuet-Higgins, M.S.(1953) Mass Transport in water waves, Phil. Trans. Roy. Soc. Series A , No. 903, Vol. 245, pp. 535-581.

Bijker, E.W., Kalkwijk, J.P.T., and Pieters, T.(1974)Mass transport in gravity waves on a sloping bottom,Proc. 14th Conf. Coastal Eng. Copenhagen, pp. 447-465.