II-216 AN OPERATOR-SPLITTING METHOD FOR INCOMPRESSIBLE FLOWS

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Introduction: The characteristic line method can be used to solve the convection dominated flows successfully. The main difficulty of this approach is the introducing of numerical damping in the convection phase. In order to limit the numerical damping, the interpolation must be done with care[1, 2]. In present calculations, the higher order interpolation functions, such as the cubic and the quadratic ones, are constructed by using the known nodal values of the adjacent elements. As the incompressible constraint can suppress the spatial oscillation, this calculation is stable.

Mathematical Model: The governing equations of the incompressible viscous flows can be expressed by the Navier-Stokes equations. In present method, the pressure is solved by deriving a Poison's equation. The momentum equation is split into the convection and diffusion equations. The convection equation is solved by the characteristic line method, the diffusion equation is solved by the explicit Galerkin's method. The details of the numerical formulations are explained in Reference [3]. For the convection equation, the liner, quadratic and cubic interpolations are investigated. For example, the cubic interpolation can be constructed by using the nodal values outside of the linear element. The formulations are

$$\phi_\alpha(\xi,\eta) = N_i(\xi)\,N_j(\eta), \quad \alpha = i + 4j - 4 \ \ (i,j = 1,2,3,4)$$

$$N_1(\xi) = -\frac{1}{16}(1-\xi)(1-9\xi^2), \quad N_2(\xi) = \frac{9}{16}(1-3\xi)(1-\xi^2),$$

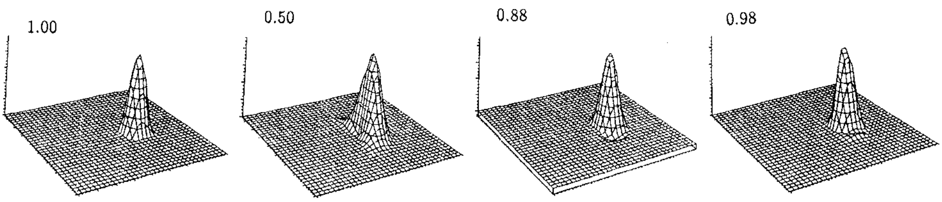
$$N_3(\xi) = \frac{9}{16}(1+3\xi)(1-\xi^2), \quad N_4(\xi) = -\frac{1}{16}(1+\xi)(1-9\xi^2) \qquad (1)$$

Example 1. The Purely Convection Flow: In order to show the accuracy of different order interpolations. A purely convection flow problem, i.e. the advection of a cosine hill in two dimensional rotating flow field, is calculated. The results are obtained at 80 time step, which is one rotation for the case of Courant number $C_r = 0.75$. Fig.1(a) is the exact solution. Fig.1(b),(c) and (d) are the results obtained by using the linear, quadratic and cubic interpolations, the maximum concentration remains about 40%, 85% and 96% respectively. For the cubic interpolation case, the filter is used to smooth the spatial oscillation.

Example 2. The Sudden Expansion Flow: The flow configuration of interest is shown in Figure 2. The calculations are carried for the Reynolds number Re = 73, 125, 191, 229, and 300. The computations are stable, i.e. no spatial oscillation occurred. The computed velocity profiles on cross sections are well in agreement with the data. The reattachment length X_R is increasing with Re numbers, as shown in Figure 3. Different interpolations for convection phase are used in the present calculations. It can be seen that the linear interpolation results in numerical damping, the reattachment length is short than the experiment data[4]. The results obtained by both the quadratic and the cubic interpolations are well in agreement with the data[4]. The results obtained by Atkens et al.[4] are also plotted in the same figure. Because the upwind scheme is used, the reattachment length is over-predicted.

CONCLUSIONS: A fractional step finite element method is used to solve the incompressible fluid flows. The convection equation is solved by the characteristic line method. The higher order interpolations, such as the cubic and the quadratic interpolations, are investigated. The sudden expansion flows are calculated. For the cubic and quadratic interpolations, the calculated velocity distributions and the reattachment length are well in agreement with measured data.

REFERENCES: [1] Forrest M. Holly et al., J. Hydraulics Div., ASCE(103), No. HY11, 1259-1277(1977).
[2] J.P.Benque et al., the Third Int. Conf. on Finite Elements in Flow Problems, June 1980, Canada, Vol.1 110-120 (1980). [3] 江, 川原, 樫山, 第四回計算力学シンポジウム, p127-132. 1990. [4] L.P. Hachman et al, Int. J. for. Numerical Methods in Fluids, Vol.4, 711-724(1984).



(a). Exact Solution (b). Linear Int. (c). Quadratic Int. (d). Cubic Int.

Fig.1 The Purely Convection Problem

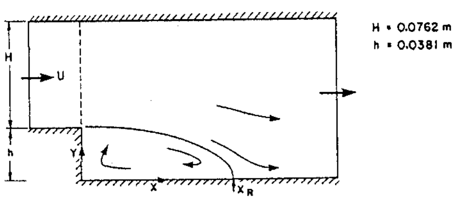


Fig.2 The Sudden Expansion Flow

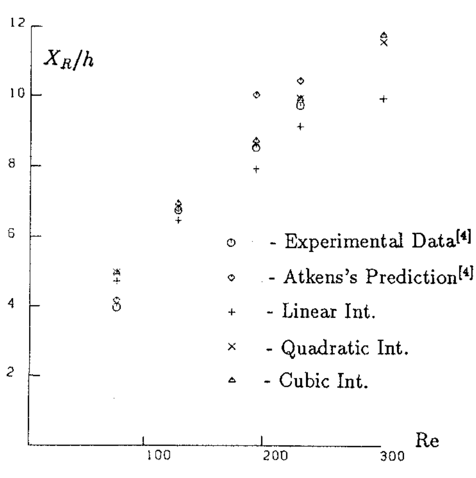


Fig.3 The Reattachment Versus Reynolds Number