I-461 ACTIVE STIFFNESS CONTROL OF CABLE VIBRATION

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Introduction: Transverse vibrations of long cables are difficult to control by dampers placed within the span. An attractive alternative approach is to control at the supports by moving them longitudinally. This time varying boundary which causes variation of transverse stiffness is herein termed "*active stiffness control*". A damping effect can be produced by this scheme. The basic idea of Chen (Ref.1) is followed and energy analysis is employed to obtain optimal control algorithm. A simple expression for equivalent damping due to active stiffness control is presented. An experiment is conducted to verify the control algorithm.

Algorithm for Single Cable: Fig. 1 illustrates a taut cable with fixed boundary at one end and longitudinally time-varying boundary at the other. The movement of the boundary, u(t), is driven by an actuator, and is assumed to be much smaller than the cable span, L. The transverse vibration is considered to be confined in one plane. Axial inertia force is assumed negligible (Ref. 2). Employing Lagrange formulation, with the normal modes (sin $n\pi x/L$) of fix-ended string as generalized coordinates, and neglecting the modal coupling terms, the governing equations of transverse motion of the taut cable under a concentrated harmonic excitation can be obtained in a nondimensionalized form as (Ref. 3):

$$\frac{d^2\tilde{y}_n}{d\tau^2} + 2n\xi_n\frac{d\tilde{y}_n}{d\tau} + n^2(1+\tilde{u}(\tau))\tilde{y}_n + \tilde{\alpha}_n\tilde{y}_n^3 = \tilde{a}_f\,\phi_n(x_0)\cos\left(\frac{\Omega}{\omega_1}\tau\right) \qquad (1)$$

where $\tilde{y}_n = \frac{y_n}{u_0}$, $\tilde{u} = \frac{u}{u_0}$, $\tau = \omega_1 t$, $\tilde{\alpha}_n = \frac{n^4\pi^2 u_0}{4L}$ and $\tilde{a}_f = \frac{2a_f}{\mu L\omega_1^2 u_0}$.

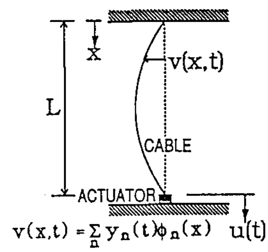

Fig. 1 Models for analysis

In the above equations, y_n is the generalized displacement of mode n; ϕ_n is the corresponding mode shape; ξ_n is the corresponding critical damping ratio; u_0 is the static elongation from unstressed condition; μ is the mass per unit length; L is the cable span; a_f is the amplitude of force; Ω is the excitation frequency; x_0 is the point of application of the external force and ω_1 is the undamped natural circular frequency of the first mode.

Energy Analysis: Energy analysis is conducted to clarify the mechanism and the optimal condition of the control. For simplicity, assume that the transverse vibration with active stiffness control, $\tilde{y}_n$, is harmonic with frequency n' which is close to the undamped natural frequency n. The control $\tilde{u}$ is also assumed to be varied sinusoidally:

$$\tilde{y}_n = \tilde{a}_{y_n}\cos n'\tau \;\; ; \;\; \tilde{u} = \tilde{a}_u\cos(sn'\tau + \gamma_{yu}) \qquad (2),\ (3)$$

where $\tilde{a}_u$, s and γ_{yu} are parameters of the control algorithm that must be selected. s is a positive constant. It is well known (Ref. 2) that parametric excitation occurs most easily when the ratio of the frequency, n, is 2. Since n' is close to n, s=2 is a logical choice for Eq. 3.

Energy production, E_p, is defined here as the integral over one steady-state oscillation cycle of $\tilde{y}_n$, of the product of generalized velocity and generalized force (any term in Eq. 1). The E_p due to the generalized force associated with $\tilde{u}(\tau)$, assuming s=2, and due to the inherent damping can be written, respectively, as :

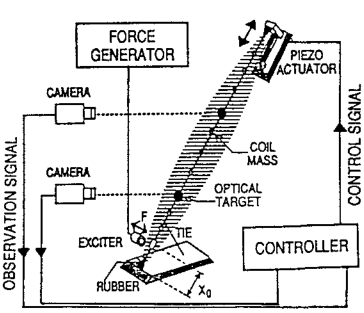

Fig. 2 Experimental set-up

$$E_p\left(n^2\tilde{u}\tilde{y}_n\right) = -\frac{1}{2}n^2\pi\tilde{a}_u\tilde{a}_{y_n}^2\sin\gamma_{yu} \;\; ; \;\; E_p\left(2\xi_n n\frac{d\tilde{y}_n}{d\tau}\right) = -2nn'\pi\,\xi_n\,\tilde{a}_{y_n}^2 \qquad (4),\ (5)$$

Equating the two expressions for E_p in Eqs. 4 and 5, the damping effect that is produced by stiffness variation can be evaluated in the form of equivalent additional damping ratio ξ_n^a:

$$\xi_n^a = \frac{1}{4}\frac{n}{n'}\tilde{a}_u \sin\gamma_{yu} \approx \frac{1}{4}\tilde{a}_u \sin\gamma_{yu} \qquad (6)$$

Eq. 6 implies that the optimum damping can be obtained when γ_{yu} is equal to 90°.

Steady State Response: In order to investigate the performance of the active stiffness control algorithm, an experiment, for the 1st mode, as shown in Fig. 2 is set up using stainless steel wire. The motion of the cable is confined in one plane. All properties of this cable are shown in Table 1. The cable is excited by a harmonic force.

If $\tilde{u}(\tau) = 0$, Eq. 1 is reduced to a well-known Duffing equation whose steady-state solutions can be analytically predicted (Ref. 2). Analytical prediction for the system with control effort can also be made by using the energy analysis in an earlier section. An equivalent damping term is employed to substitute the control effects so the solutions for Duffing equation can be used by replacing ξ_1 by $\xi_1 + \xi_1^a$.

The experiment is divided into two parts. For the first part, $\tilde{a}_f$ is kept constant while Ω is varied in the neighborhood of ω_1. The steady-state response with and without active stiffness control are shown in Fig. 3. With active stiffness control, $\tilde{a}_u$ and γ_{yu} are fixed at 2.27% of initial elongation and 90° for all excitation frequencies Ω. In both cases, the experimental results are well predicted by the analytical solutions (thin curves). Large reduction in response amplitude even with small $\tilde{a}_u$ confirms the effectiveness of active stiffness control. The second part concerns with the effects of $\tilde{a}_u$ and γ_{yu} on the controlled response. The resonant peaks for different $\tilde{a}_u$ with optimum phase γ_{yu}, and for different phase γ_{yu} with fixed $\tilde{a}_u$, are shown in Figs. 4 and 5, respectively. The experimental results confirm that higher additional damping can be gained at higher $\tilde{a}_u$, and the damping is optimum when γ_{yu} is 90°. In both cases, very good agreement with prediction is obtained.

Another important feature of the active stiffness control is that it can be applied to any low-order mode. Control of 2nd mode with various values of the amplitude $\tilde{a}_u$ is experimentally studied under the optimal condition (γ_{yu} = 90°). The experiment and the prediction, as shown in Fig. 4, are in very good agreement and the effectiveness of the active stiffness control is confirmed.

Table 1 Physical parameters of cable

Type/Material : Stainless wire rope, SUS 304 (JIS)		
Metallic cross-sectional area (A):	2.055×10^{-3}	cm^2
Rope modulus of elasticity (E) :	1.74×10^{6}	kg/cm^2
Pre-tension (T) :	83	Newtons
Span (L) :	208	cm
Mass per unit length (μ) :	0.07	kg/m
Fundamental natural frequency	8.28	Hz
Critical damping ratio of Mode 1	1.60×10^{-3}	
Critical damping ratio of Mode 2	1.89×10^{-3}	

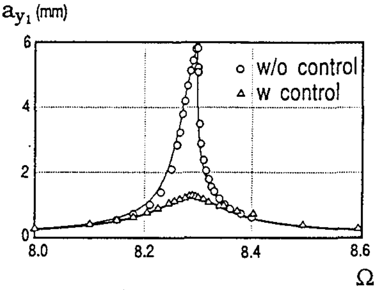

Fig. 3 Amplitude of harmonic response with and without active stiffness control

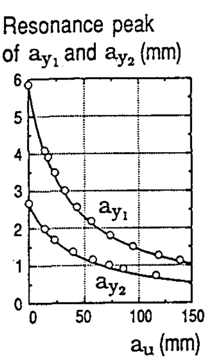

Fig. 4 Effect of control amplitude, a_u on the resonance peak amplitudes

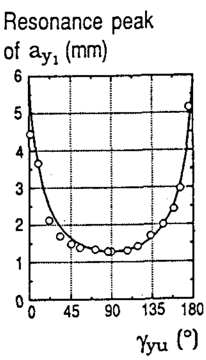

Fig. 5 Effect of phase, γ_{yu} on the resonance peak amplitude

References: 1) Chen, J.C., J. of Spacecraft and Rockets, Vol. 21, No. 5, 1984., 2) Nayfeh, A. H. and Mook, D. T., Nonlinear Oscillations, 1979., 3) Warnitchai, P., Nonlinear Vibration and Active Control of Cable-Stayed Bridge, D.Eng. Thesis, U. of Tokyo, 1990.