I-459 ACTIVE TENDON CONTROL OF A CABLE STAYED GIRDER

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1. FRAMEWORK

Active control in civil engineering is a challenging field that aims at building safer and lighter structures. In this paper, active control of cable-stayed bridges subject to a vertical harmonic force is analytically and experimentally studied.

The **analytical model** is a simplified version of the general model of cable-stayed bridge vibration given in [1]. The interaction between the girder and the cable vibrations, a phenomenon known as internal resonance [2], is taken into account through a global-local interaction. 'Global' vibration refers to the motion of the girder, pylon, and cables as one assemblage, the cables behaving like elastic tendons. 'Local' vibration on the other hand refers to the transverse oscillation of a cable between fixed supports. Both use the modal approach. The analysis models the girder vibration under the finite displacement theory, whereas geometric non-linearity and sag considerations are allowed for the cable. Using Lagrange equations, this results in linear and quadratic couplings between the global and local vibrations, while quadratic and cubic couplings exist within the cable.

The **control loop**, aiming at damping the girder vertical motions, is based on a velocity feedback scheme [3] using the mid-span girder strain as the girder tip displacement indicator and producing a control force applied to the girder tip through a dynamic cable axial elongation. The sensor, a strain gage circuit, and the controller, a piezoelectric actuator, are not collocated. A phase shifter is used to obtain the girder velocity from the strain signal.

The **experimental model** is composed of a uniform beam with onefixed end and supported by a stay cable. It was carefully designed to replicate faithfully the principal dynamic characteristics of actual cable-stayed bridges. The actuator is integrated to the structure by placing it at the cable top anchorage.

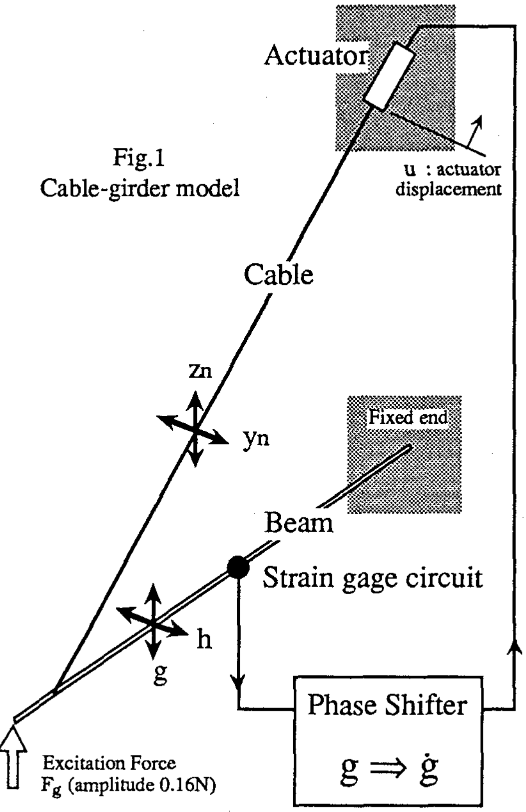

Fig.1
Cable-girder model

The global motion is defined by vertical and horizontal generalized coordinates g and h. The local motion is defined by vertical and horizontal generalized coordinates z_n and y_n. The **equations of motion** describing the system and driving the actuator motion are given along with Fig.1, a summary of the setting. Of the three effects of the actuator motion, namely active tendon force, active stiffness variation force and sag-induced force, only the first one is considered in the remainder of the paper.

$$m_g\ddot{g} + 2\xi_g m_g \omega_g \dot{g} + m_g \omega_g^2 g + \sum_n \alpha_{gn}\ddot{z}_n + \sum_n \eta_{gn}(z_n^2+y_n^2) + P_g u + S_g \ddot{u} = F_g(t)$$

$$m_h\ddot{h} + 2\xi_h m_h \omega_h \dot{h} + m_h \omega_h^2 h + \sum_n \zeta_{hn}\ddot{y}_n = 0$$

$$m_{zn}\ddot{z}_n + 2\xi_{zn} m_{zn}\omega_{zn}\dot{z}_n + m_{zn}\omega_{zn}^2 z_n + \alpha_{gn}\ddot{g} + 2\eta_{gn} g z_n + \sum_k 2\beta_{nk} z_n z_k + \sum_k \beta_{kn}(y_k^2+z_k^2) + \sum_k \nu_{nk} z_n (y_k^2+z_k^2) + R_n u z_n + T_n \ddot{u} = 0 \quad n=1, 2, 3..$$

$$m_{yn}\ddot{y}_n + 2\xi_{yn} m_{yn}\omega_{yn}\dot{y}_n + m_{yn}\omega_{yn}^2 y_n + \zeta_{hn}\ddot{h} + 2\eta_{gn} g y_n + \sum_k 2\beta_{nk} y_n z_k + \sum_k \nu_{nk} y_n (y_k^2+z_k^2) + R_n u y_n = 0 \quad n = 1, 2, 3..$$

$$u = G(g)_{90^\circ \text{shift}} \cong G\left(\frac{\dot{g}}{\omega_g}\right)$$

2. EXPERIMENT

Many parameters such as global mode shapes, damping ratios, natural frequencies, or the elastic modulus of the cable were measured directly on the model.

By varying the cable tension, three important coupling patterns were investigated analytically and experimentally with and without control. For each case the frequency response was obtained for a vertical and harmonic excitation of the girder. Analytical predictions were obtained through a perturbation technique applied to the above equations, keeping only the relevant coupling terms. Table 1 summarizes the identified frequencies for each case (in Hertz).

Table 1 : Experimental case ctudies

Type of motion	Global mode	Quadratic coupling	Linear coupling
Global vertical	19.82	19.81 (2:1 with 9.385 and 9.63)	19.82 (1:1 with 19.37)
Global horizontal	*	9.385 (1:1 with 9.63)	*
Local horizontal	*	9.63	*
Local vertical	*	•	19.37

* untuned • restrained

3. GLOBAL MODE

For reasonable levels of the gain, the control scheme proved to be **very effective** and the analytical model very accurate (see Fig.2). But **spillover effects** were observed for higher gains due to the presence of unfiltered frequencies near the frequency under control.

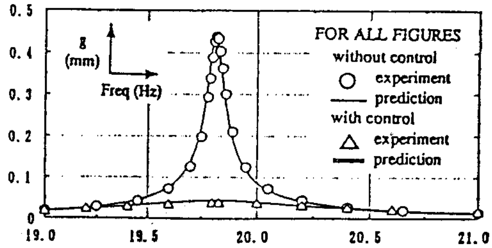

Fig.2 Pure global mode

4. QUADRATIC COUPLING

As for the case with the global mode only, the control scheme and the analytical predictions agreement proved to be **satisfactory** as seen in Fig.3. The horizontal motions of the cable and the girder induced in the frequency range stressed in Fig.3, a typical **nonlinear phenomenon**, are not presented here for the sake of brevity, but they were completely suppressed by the control action.

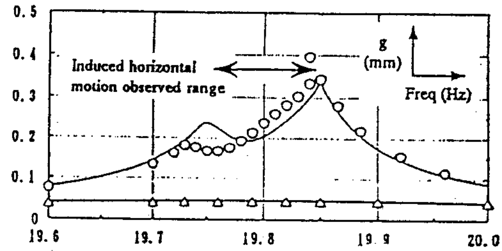

Fig.3 Quadratic coupling : girder part

5. LINEAR COUPLING

The frequency response of the system without control exhibits **a classical pair of principal modes** that can be termed local-dominated (around 17.25Hz) and global-dominated (around 19.8Hz) when looking at the kinetic energy distribution between the girder and the cable. The analytical model agrees satisfactorily. When the control scheme was applied, although **the global dominated peak was significantly reduced**, in accordance with the predictions, **the local dominated peak was scarcely affected** (see Fig.4). To investigate this discrepancy, separate tests at the local dominated frequency for several gain levels were conducted, the results being in Table 2, showing that some **modal properties of the model were modified**. Tip motion is the girder tip actual displacement over the one computed from the mid span strain signal. This ratio is assumed to be unity if the real mode assumption is verified. This is not exactly the case. Nevertheless, it seems that amplitude wise the girder modal properties were not affected. Cable/girder is the amplitude ratio between the cable and the girder motions. It seems that the control action provokes a greater energy storage in the cable. Phase is the phase lag between the cable motion and the mid span strain. But it is a good indicator of the phase lag between the girder tip and middle as the cable is excited relatively far from its natural frequency, hence keeping almost the same phase lag with the girder tip where the excitation force acts. Therefore, we have here some concrete evidence that the **girder mode shape became complex.**

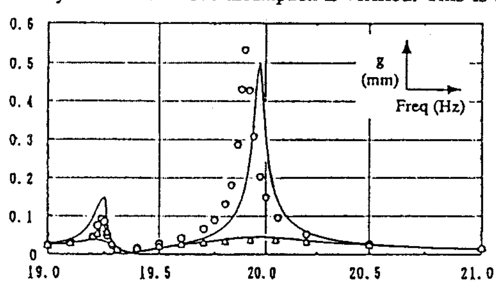

Fig.4 Linear coupling : girder part

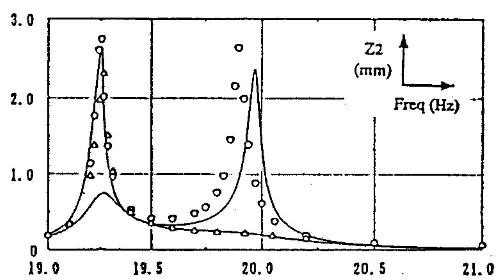

Fig.4 Linear coupling : cable part

This modal distortion under the conjunction of linear coupling and the control action was not entirely explored here, but may anyway be an obstacle in practical implementations of active control, as widely recognized standard control theories do not take it into account.

Table 2 : Modal distortion

Gain level	Tip motion	Cable/girder	Phase
0	0.847	34.0	17.3
	(1)*	(18.3)	(8.8)
0.412	0.843	39.8	-20.7
	(1)*	(19.8)	(9.5)
0.762	0.840	45.2	-38
	(1)*	(19.2)	(9.2)
1.483	0.843	17.6	-58.7
	(1)*	(17.6)	(9.1)

( )* assumption ( ) prediction

6. ACKNOWLEDGEMENTS

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7. REFERENCES

[1]Warnitchai, P. (1990)"Nonlinear vibration and Active control of Cable-stayed bridges", Doctoral Dissertation, U. of Tokyo. [2]Maeda, K., M. Yoneda, and Y. Maeda (1984) Proceedings of 12th IABSE Congress, Vancouver, pp747-754. [3]Forward, R. L. (1979) Applied Optics, Vol.18, No5, March, pp. 690-697. Forward, R. L. (1981) Journal of Spacecraft and Rockets, AIAA 8.1-4018, Vol.118, No1, Jan.-Feb., pp 11-17.