I-445

EFFICIENT SUPPRESSION OF VIBRATION WITH EXPLICIT TREATMENT OF ACTUATOR'S LIMIT

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INTRODUCTION:

In the presence of control effort penalty in the objective function of the classical optimal linear regulator problem, the maximum capacity of an actuator cannot be utilized fully. To utilize the available control energy efficiently, the present study proposes a new control algorithm with the control effort penalty removed from the objective function. As a substitution, an inequality constraint is imposed on the control effort magnitude with practical limits of an actuator as bounds. A similar objective function with final time unspecified and final states specified has been studied by Wonham and Johnson [1964] which led to singular control. On the contrary, the present study proposes an objective function or performance index with fixed final time and free final states which leads to a control strategy which is similar to bang-bang control.

FORMULATION:

To facilitate the presentation, a damped SDOF system will be considered, however the generality of the proposed algorithm will not be violated. The equation of motion in the state-space form can be written as

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -\omega^2 x_1 - 2\xi\omega x_2 + u + u_e \tag{1a,b}$$

where $u = f/m$; $u_e = f_e/m$; $x_1 = x$; $x_2 = \dot{x}$ and f, f_e are control force and disturbance force respectively. The performance index (PI) is defined as

$$J = \frac{1}{2}\int_0^T (x_1^2 + q\, x_2^2)\, dt \tag{2}$$

where q is a weighting factor and T is a specified final time. According to the Minimum Principle of Pontryagin, the Hamiltonian can be obtained as

$$H = \frac{1}{2}(x_1^2 + q\, x_2^2) + \lambda_1 x_2 - \lambda_2 \omega^2 x_1 - 2\xi\omega\lambda_2 x_2 + \lambda_2(u + u_e) \tag{3}$$

The costate equations are obtained by taking the first derivatives of H

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = -x_1 + \lambda_2\omega^2 \qquad \dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = -q x_2 - \lambda_1 + 2\xi\omega\lambda_2 \tag{4a,b}$$

The optimal control u* that minimize J should also minimize H, which leads to the optimal control law $u^* = -\bar{u}\,\mathrm{sgn}(\lambda_2)$ (5)

where $\bar{u}$ is the bound of u. By variational calculus, the boundary condition at the final end can be derived as $\lambda_1(T) = \lambda_2(T) = 0$ (6)

If we identify T as a time interval between two successive switching times of actuator and T is very small we can expect the sign of λ_2 is constant in this small interval. The PI can be rewritten in instantaneous form as

$$J = \frac{1}{2}[x_1^2(T) + q\, x_2^2(T)] \tag{7}$$

The Hamiltonian and the costate equations reduced to

$$H = \lambda_1 x_2 - \lambda_2\omega^2 x_1 - 2\xi\omega\lambda_2 x_2 + \lambda_2(u + u_e) \tag{8}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = \lambda_2\omega^2 \tag{9a}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = \lambda_1 + 2\xi\omega\lambda_2 \tag{9b}$$

And the boundary conditions at the final end becomes

$$x_1(T) = \lambda_1(T) \quad q x_2(T) = \lambda_2(T) \tag{10}$$

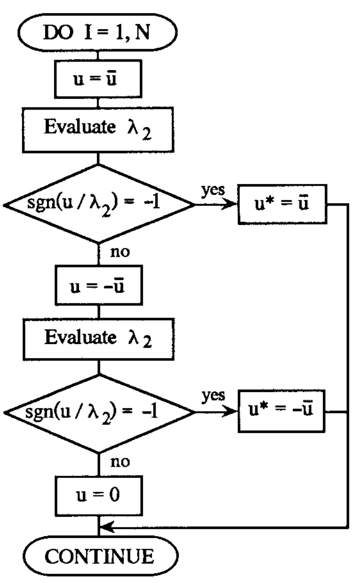

Fig.1: Control Strategy

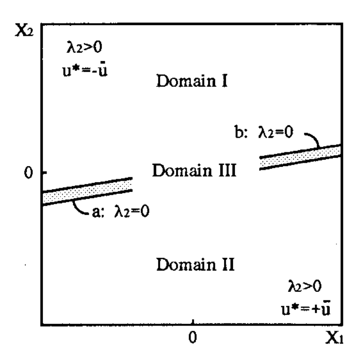

Fig.2: Domains of control activity

In the numerical examples, the instantaneous PI will be adpoted as objective function.The control stategy is summarized in Fig.1. The inactive control intervals could be explained by referring Fig.2. Lines a and b are obtained by assigning $u=\bar{u}$ and $u=-\bar{u}$ respectively. There are three domains of control activity. If the initial condition (X_1 , X_2) is located in domain I, the optimal control can be achieved by applying $u^*=-\bar{u}$, while in domain II, by applying $u^*=\bar{u}$. In domain III, the optimality cannot be achieved by applying $u^*=\bar{u}$ or $u^*=-\bar{u}$, and this study inactivates the control action during the intervals with initial conditions located inside domain III.

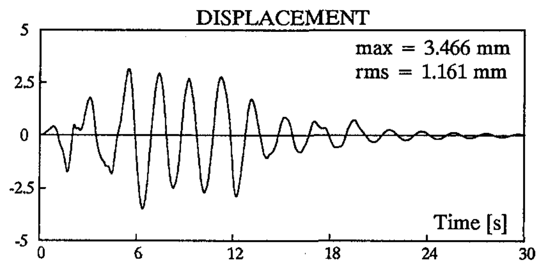

Fig.3: Uncontrolled response

NUMERICAL EXAMPLE:

A damped SDOF system with mass, damping and stiffness constants 1 Ns^2/mm, 0.316 Ns/mm (5% damping) and 10 N/mm respectively, is subjected to random base excitation, 2% scaled down El Centro 1940, NS component earthquake time history. To demonstrate the effectiveness of the new optimal control method and the superiority to the classical optimal linear regulator method with performance index

$$J = \frac{1}{2}\int_0^T (x_1^2 + q\,x_2^2 + r\,u^2)\,dt \tag{11}$$

the given system are simulated by both control algorithms and the results are presented in Figs.3 to 5. It can be observed that the classical optimal control law requires extremely higher maximum control force, i.e. 125% higher as compared to that of the new optimal control law. To measure the effectiveness in utilizing the available control energy, an efficiency factor is introduced

$$\beta = \int_{t0}^{t1} u^2\,dt \Big/ \int_{t0}^{t1} u_{max}^2\,dt \tag{12}$$

where u_{max} is the maximum control force magnitude during the period of simulation.

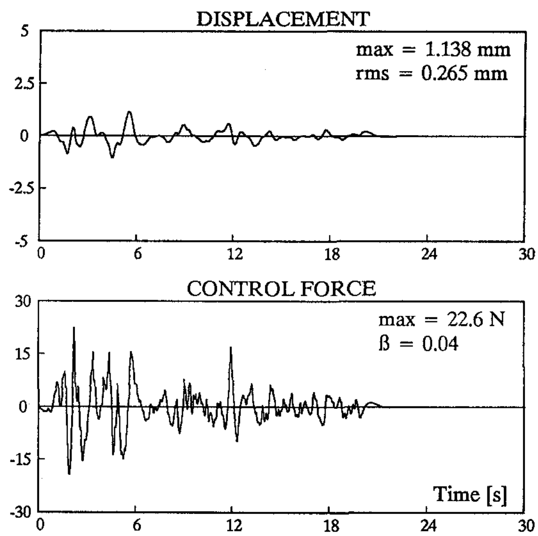

Fig.4: Classical optimal control (q = 1.0, r = 0.065)

CONCLUDING REMARK:

The newly proposed optimal control law has been demonstrated to be superior in efficient use of the available control energy and in suppressing the vibration level due to slowly varying sinusoidal disturbance and random disturbance.

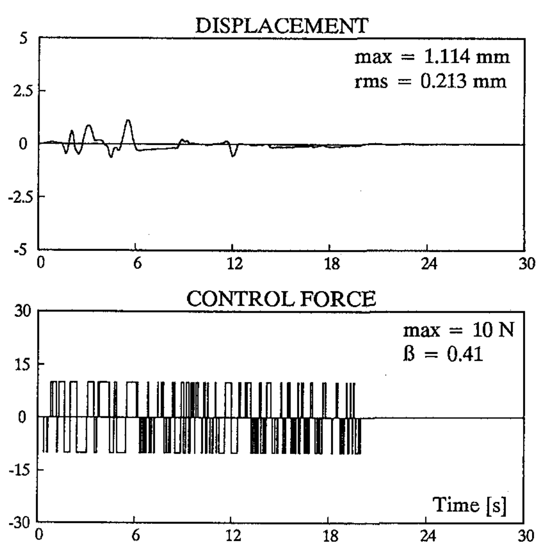

Fig.5: New optimal control (q = 1.0)

REFERENCE:

Wonham, W.M., Johnson, C.D., J.Basic Engineering, Trans. ASME, March 1964