I-444 ACTIVE REDUCTION METHOD OF VIBRATION OF STRUCTURES BASED UPON CLASSICAL VARIATIONAL PRINCIPLE

Earthquake Research Institute, Univ. of Tokyo, Member Hiromichi HIGASHIHARA

1.Introduction

The present study tries to apply the well established mathematical theory of optimal control to the reduction problem of seismic vibration of civil engineering structures. Employing the classical formulation and explicitly considering the power limits of actuators, it develops efficient control criteria; accordingly, these formula have physical realizations. Existence of physical realizations is an indispensable condition for engineering models of control problem, and any numerical simulations lacking them has little value.

A physical model which represents the proposed mathematical model is then constructed and the appropriateness of the obtained results is verified by means of laboratory experiments. Some results are presented elsewhere(1) and only conceptual basis of the study is summarized herein.

2.Theoretical Background

We divide the prescribed time interval into successive sequence of small subintervals and consider a control process such that control variables hold constant in each subinterval. Then the problem is how to determine feasible and in some sense optimal values of the control variables to be allocated at the beginning of every subinterval. This approximation does not lose universality from the discussion.

Mathematical theory of control has been well established as an extension of the classical variational calculus, or equivalently, as the theory called the Pontryagin's maximum principle. It generally deals with three types of mathematical expressions simultaneously: a functional called objective function, equations of motion of the total system including the structures and the actuators, and some algebraic equations called constraints.

The objective function is defined as an integral of response variables over the prescribed time interval. If its integrand is selected as a quadratic form of the response variables, subsequent discussions remains within linear differential equations; various useful results are obtainable.

3.Linear Regulator Formulations

Independently of the progress of the mathematical theory, feedback control technique has been highly developed in the industrial practice of automatic control. It is widely recognized that appropriate feedback of signals of response variables improves the dynamic properties such as damping ability of structures.

Classical mathematical theory has been reformulated from this viewpoint and a new method has been developed which is called the linear optimal regulator theory. In this method, the integrand of the objective function takes a quadratic sum of response variables and control variables. Control criteria of feedback type is then obtainable. Unfortunately this procedure is generally troublesome; i.e., in order to determine the feedback gain, one must solve a highly nonlinear differential equation over the whole time duration of interest in the backward direction. Nevertheless, quite a lot of works have been devoted to this problem and many useful methods are available.

4.Instantaneous Optimality

Conventional works have applied the regulator theory to the suppression problem of seismic vibration of structures while introducing a very strong approximation: the instantaneous objective function. This approximation defines the objective function as an integral over very small time interval next to every time point at which the control force is applied.

The present study follows this approximation, but it must be pointed out that this approximation has eliminated an intractable and yet potentially fruitful issue of the control theory; it enables the control criteria only to take the instantaneous situation of vibration into consideration and to pay no attention to its long term course. Consequently, the synthesis procedure, the central technique of constructing the control program, has disappeared at all. In other words, this approximation is content with the substitution of the global optimality with a patchwork of locally optimal paths; accordingly, the most outstanding product of the classical control theory such as preliminary loading of reactions which will cancel the future action of violent seismic motions.

Under the assumption of instantaneous objective function, the solution procedure of the regulator theory is extremely simplified; a control criteria of feedback type is obtainable in a straightforward manner without solving any differential equations.

5.Direct Use of Classical Variational Method

Linear regulator method is very powerful, but it is worth emphasizing that its solution is only optimal under the condition that the ability of the actuators is so sufficiently large as to always be able to provide the control force requested by the control criteria. This is not the case if one tries to use ordinary actuators for suppression of vibrations of large civil engineering structures under the action of strong earthquakes.

A basic standpoint of the present study is that the limitation of the loading ability of the actuators must be treated explicitly. It is possible to interpret the application of linear regulator theory to aseismic control of vibration as a transformation of the constraints of the power of actuators into the penalty term of control forces added to the quadratic form defining the objective function. But this seems to be a poor trial.

First, how is the relative weight of the penalty term determined in the objective function ? There is no reasonable unique criterion of choice, and it is tentatively tried and modified in the practice of simulation. If the weight of the penalty is too small, the obtained solution violates the capacity constraint of the actuators, while if this weight is too large, reduction of the response would remain poor. Anyway one must intuitively and heuristically find appropriate weight for the individual case; i.e., optimality remains unsolved theoretically.

Secondly, feedback type control criteria with constant gain almost always direct the actuators their intermediate power and scarcely request their maximum ability. This clearly indicates that feedback type control criteria cannot be efficient for the present problem because optimal control force must be identical at every instance to the extremals of the actuators (switching theorem). This tendency grows as the ratio of the power of the actuators against the scale of structures decreases or as the subintervals between two adjacent operation time points shorten. In the application to aseismic countermeasure of civil engineering structures, these conditions are true; accordingly, deficiency of feedback type control criteria is expected to be large.

Another nonnegligible constraint which physical actuators carry is the existence of the delay time of their mechanical motion. Total delay time is the sum of individual delay time of the elements of the control system: sensors such as accelerometers, amplifiers, A-D converters, computation circuit of the control criteria, D-A converters and the actuators themselves.

In the present study, direct drive method with linear motors has been adopted because of their very quick responsiveness. Even in this situation, the rise time of mechanical reaction of the motors is far larger than the sum of the remaining electrical reaction times. Roughly speaking, time delay problem of control system is therefore that of the mechanical devices which provide the reactions. Delay characteristics of popular motors are investigated in detail and accurate data are available. According to these data, delay properties of the standard direct drive linear motors are describable in terms of a linear first order differential equation, so long as the displacement of the shafts of the motors remains within some limited range. The classical variational calculus can cover this problem, yielding a little more complicated equation without losing the validity of the logic which derived the control criteria in case of nondelayed system. In this sense, the method of direct use of the classical variational calculus has wide applicability.

(1) Indrawan,B. and Higashihara,H. ; Efficient suppression of vibration with explicit treatment of actuators' limit , Proc. 46th Annual Conference, JSCE, 1991