I－25 A Homogenization Method for Solids Containing Multiple Cracks

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I. Introduction It is well known that when the crack density is small, or the cracks are of dilute distribution, the non-interaction solution is adequate for most engineering problems. However, the interaction effect can not be neglected when the crack density is high. Various kind of approximation methods have been proposed to take the interaction effect into account. In case of isotropic crack orientation statistics, the well-known Self-Consistent Method(SCM) gives the prediction of vanishing elastic stiffness at a crack density of $\rho = Na^3 = 9/16$ for 3D problems and $\omega = \pi N a^2 = 1$ for 2D problems. This is theoretically unacceptable and is not in agreement with experimental results. The prediction of SCM can not be used for large crack densities.

A new method is proposed here to evaluate the average values of cracked materials. The method is based on the non-interaction solution, which is also called Taylor's solution in our present discussion. The Taylor's solution is used in our evaluation method and the resulting average fields can be expressed in closed form. It is found that the limiting case of our evaluation process corresponds to the SCM solution and the results of the first cycle evaluation is very close to the Differential Scheme(DS) solution. The proposed solution, which is also in very good agreement with experimental results, could be used as an approximation for high crack densities.

II. General Consideration Let's consider a Representative Volume Element(RVE) which contains a large number of cracks in a given geometry statistics. The general stress-strain relation can be written in the following form (Horii and Nemat-Nasser,1983)

$$\bar{\epsilon}_{ij} = C_{ijkl}\bar{\sigma}_{kl} + \frac{1}{2V}\int([u_i]n_j + [u_j]n_i)dS \qquad (1)$$

where n_i denotes the crack normal, $[u_i]$ denotes the displacement jumps across the crack surface and V is volume of RVE. The second part in (1) is the crack-induced additional strain which is derivable from a knowledge of the displacement jump fields. However, it is not easy to solve the general interaction problem for the field of displacement jumps. Approximation has to be performed.

To the Taylor's approximation, a crack is embedded in the isotropic elastic solid (fig.1a) to feel the average field while other cracks are ignored (fig.1b). The additional strain can be calculated using (1). Each crack is treated in the same way and the summation is made to give the average field (fig.1c,d). It seems that the additional strains are underestimated since the presence of other cracks tends to reduce the stiffness of the "effective solid" (fig.1a). The Taylor's solution might overestimate the stiffness hence underestimate the compliance of the material.

To the present solution, a single crack is embedded in the matrix to feel the Taylor's average field, while other cracks are neglected (fig.1e). The additional strains are calculated and the contribution of each crack is summed to evaluate the new average field (fig.1f,g). The approximation of additional strains is greatly improved because the presence of other cracks is reflected through the Taylor's average field. The same process can be repeated, e.g., the second-cycle solution can be obtained based on the proposed solution and the third-cycle solution can be obtained based on the second-cycle solution. One obvious suspicion is that as the previous average field is weakened by the present embedding process, higher-cycle solutions other than the proposed solution would overestimate the interaction effect.

We find that the nth-cycle solution converges to the SCM solution as the limiting case when $n \to \infty$. This can be seen in examples in the next section.

III. Examples When cracks are randomly oriented in the 2D matrix and remain open, the macroresponse is elastic and the average field can be described by $\bar{E}$, $\bar{G}$ and $\bar{\nu}$, the effective Young's modulus, shear modulus and Possion's ratio. The Taylor's solution results in

$$\bar{E}/E = \bar{\nu}/\nu = 1/(1+\omega)$$

The present solution is obtained as

$$\frac{\bar{E}}{E} = \frac{\bar{\nu}}{\nu} = \frac{1}{1+\omega(1+\omega)}$$

The limit of the nth-cycle solution is

$$\lim_{n\to\infty}\frac{\overline{E}}{E}=\lim_{n\to\infty}\frac{1-\omega}{1-\omega^{n+1}}=\begin{cases}1-\omega & if\ \omega<1\\ 0 & if\ \omega>1\end{cases}$$

which is just the solution of SCM. The Taylor's, the proposed, SCM and other higher-cycle solutions are shown in fig.2 in terms of normalized Young's moduli. Also shown are the results by Kanaun (1980) and the experimental results by Vavakin and Salgnik[cited in Kanaun(1980)]. The proposed solution agrees well with the experimental result.

In case of unidirectionally distributed open cracks, the Taylor's solution is

$$\frac{\overline{E}}{E}=\frac{1}{1+2\omega}\quad;\quad\frac{\overline{G}}{G}=\frac{1+\nu}{1+\nu+\omega}$$

The proposed solution is obtained as

$$\frac{\overline{E}}{E}=\frac{1}{1+\omega a_1}\quad;\quad\frac{\overline{G}}{G}=\frac{1+\nu}{1+\nu+\omega b_1}$$

where

$$a_1=\sqrt{\frac{1+2\omega}{2}(\sqrt{1+2\omega}+1+\omega)};\quad b_1=\sqrt{\frac{1}{2}(\sqrt{1+2\omega}+1+\omega)}$$

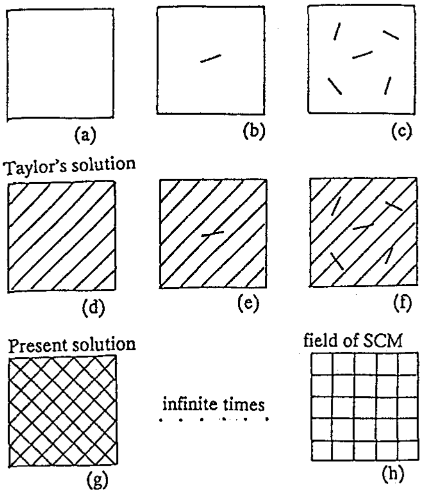

Fig.1 Evaluation Process

Similarly, the nth-cycle solution can be obtained and it converges to SCM solution as $n\to\infty$. The results of our solution as well as the DS solution (see Hashin,1988) are given in fig.3. The proposed solution agrees well with DS solution.

IV. Conclusion The proposed solution of average field of cracked materials, which is based on the Taylor's solution, appears to be more realistic approximation since the results are in good agreement with experimental data and other approximation methods. Besides, this method is conceptually simple and mathematically easy. This can be used as an approximation method for the solids containing multiple cracks.

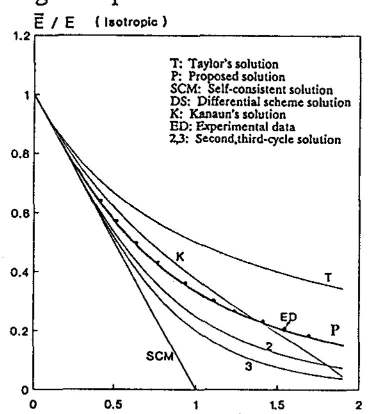

Fig.2 CRACK DENSITY

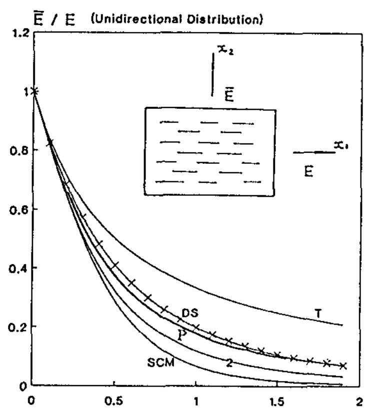

Fig.3 CRACK DENSITY

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