II-351

FRICTION FACTOR UNDER WAVE OF PERMANENT TYPE AND CURRENT

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ABSTRACTS

A rotational stream function wave theory is proposed with the application of the Finite Fourier approximation to study the interaction between wave and current. The method is different from Tanaka(1989) which is a modified version of Dean's stream function. The numerical results in terms of friction factor are then compared with several conventional eddy viscosity models.

I. *STREAM FUNCTION THEORY*

The problem to be considered herein is that of the two-dimensional, periodic wave of permanent form. The method is different from Tanaka(1989) in the followings.

The method assumes first the wave height is given while Tanaka used the measured wave profile. Second, it can be used for any value of water depth because of the inclusion of the term cosh(jkD) in the coefficient which was not in Tanaka's expression. Third, it assumes the linear vertical distribution of the shear stress for steady current while Tanaka assumed a constant shear stress.

By taking a frame of reference moving with the wave celerity, the problem is reduced to one of steady flow. The rotational stream function(with the application of the eddy viscosity model) is assumed in the form of:

$$\psi = B_o(z - z_o) + \psi_c + \sum_{j=1}^{n} \frac{B_j \sinh jk(z-z_o) \cos(jkx)}{\cosh(jkD)} + \frac{K_o k}{2\omega} \sum_{j=1}^{n} \frac{B_j}{\cosh(jkD)\, R_{jo}} \{F_j\} \tag{1}$$

Where $B_o, B_1, B_2, , , B_n$ = Unknown constants for a particular wave
ψ_c = Stream function representing the steady current given by

$$\psi_c = \frac{[\{\ln(z/ez_o) - (0.5z - z_o)/D\}z - \{\ln(e^{-1}) + 0.5z_o/D\}z_o]}{[\{\ln(D/ez_o) - (0.5D - z_o)/D\} - \{\ln(e^{-1}) + 0.5z_o/D\}z_o/D]} U_c \tag{2}$$

U_c = Depth averaged steady current
z_o = Roughness parameter, e = 2.718(Base of the natural logarithm)
D = Mean water depth
K_o = κu_{*wc} (κ=0.4, u_{*wc}=Max. shear velocity of the combined flow)
k = Wave number
ω = Angular frequency
$R_{jo} = \mathrm{Ker}^2\xi_{jo} + \mathrm{Kei}^2\xi_{jo}$, $\xi_j = 2(j\sigma z/K_o)^{1/2}$, $\xi_{jo} = 2(j\sigma z_o/K_o)^{1/2}$

$$\begin{aligned} F_j = {} & \xi_j(\mathrm{Ker}'\xi_j \mathrm{Kei}\xi_{jo} - \mathrm{Kei}'\xi_j \mathrm{Ker}\xi_{jo})\cos(jkx) \\ & - \xi_j(\mathrm{Ker}'\xi_j \mathrm{Ker}\xi_{jo} + \mathrm{Kei}'\xi_j \mathrm{Kei}\xi_{jo})\sin(jkx) \\ & + \xi_{jo}(\mathrm{Kei}'\xi_{jo} \mathrm{Ker}\xi_{jo} - \mathrm{Ker}'\xi_{jo} \mathrm{Kei}\xi_{jo})\cos(jkx) \\ & + \xi_{jo}(\mathrm{Kei}'\xi_{jo} \mathrm{Kei}\xi_{jo} + \mathrm{Ker}'\xi_{jo} \mathrm{Ker}\xi_{jo})\sin(jkx) \end{aligned}$$

(Ker and Kei are Kelvin functions)

Eqs. 1 and 2 are solved under the following boundary conditions:
$\psi = 0$ at $z = z_o$, and $\psi = \psi_\eta$ at $z = \eta$(water surface)
$p = 0$ at $z = \eta$ (Bernoulli equation)

II. *SOLUTION TECHNIQUE*

The solutions are solved as shown in diagram 1.

III. *RESULT DISCUSSIONS*

The comparisons are made in terms of the friction factor with several models with the modification on the dispersion relation as:

$$\omega = \sigma + k\tilde{U}$$

$$\sigma^2 = gk\tanh(kD)$$

$$\tilde{U} = \frac{ku_{*c}/\kappa}{\sinh(2kD)}\left\{\int_{z_0}^{D} \ln(z/z_0)e^{2kz}dz + \int_{z_0}^{D}\ln(z/z_0)e^{-2kz}dz\right\}$$

The friction factor is defined as follow:

$$\tau_{wcm} = \frac{\rho}{2} f_{wc} U_0^2$$

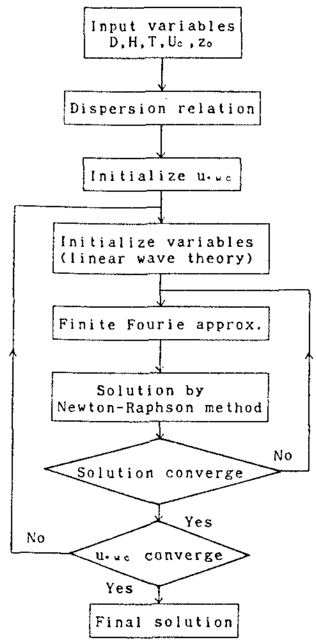
where τ_{wcm} = Maximum bottom shear stress

U_0 = Velocity amplitude by the linear wave theory

Fig. 1 is for the case of wave alone. The present study gives results somwhat similar to those by the GM(Grant and Madsen,1979),TS (Tanaka and Shuto,1984), and CJ2 (Christoffersen and Jonnson, Model 2 ,1985) models, although only the present study includes the non-linear effect. For a small wave heights, however, the GM and CJ1 (Christoffersen and Jonsson, Model 1,1985) models predict rather high values due to an arbitrary choice of a constant of length scale in the GM model and a constant eddy viscosity assumption in the CJ1 model respectively.

Fig. 2 is for the case of wave-current co-directional motion. Close agreements are found between the TS model without and the present study with the non-linear effect. On the other hand, the results predicted by GM, CJ1, and CJ2 models are lined in the same ranges with lower values than the TS model and the present study. This is because of the differenc in the steady current formulation. However, this requires the experimental verification.

Diagram 1



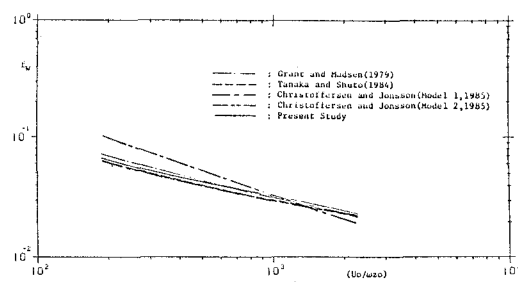


Fig. 1 Wave alone
(D=10 m, T=8 sec, H=0.20-5.00 m, zo=0.001 m)

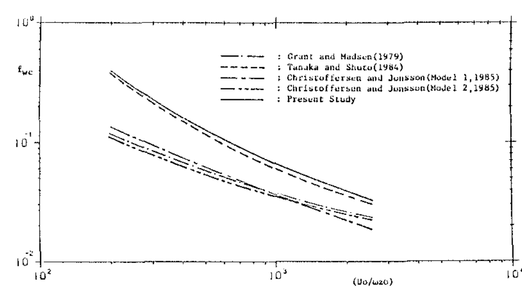


Fig. 2 Wave and Current Co-direction
(D=10 m, T=8 sec, H=0.20-5.00 m, zo=0.001 m, Uc=1.00 m/s)

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