# II-312 THE INTERACTION OF WAVES AND PORO-VISCO-ELASTIC SOIL BOTTOM

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## 1. INTRODUCTION

When water wave propagates over deformable seabed wave energy is dissipated by the viscous friction in the seabed, on the other hand the liquefication and failure of seabed may occur due to the periodic wave loading. In this paper an analytical solution for periodic waves propagating over a porous deformable bottom is presented for the case where the soil under the seawater is regarded as a poro-visco-elastic material of infinite depth.

The effects of the viscosities of both water and soil are included in the theoretical analysis. A dispersion relation of wave propagation which is modified by the interaction of wave and bottom is derived, from which the rates of wave attenuation are calculated. Exact solutions are obtained for calculating the distributions of the pore-water pressure, the displacements of soil particles and the stresses in the poro-visco-elastic bottom.

The calculated results indicate that the responses of soil bottom are strongly influenced by the permeablity and stiffness of soil, the compressibility of the pore-water as well as the viscosity of soil.

## 2. THEORETICAL ANALYSIS

### 2.1 Governing equations

Here we will study the case where a two-dimensional wave propagates over an isotropic poro-visco-elastic soil bottom of infinite depth and length. It is assumed the wave motion is laminar and the wave is of small amplitude. Then we employ the following laminar Navier -Stokes equations for the water wave:

$$\frac{\partial u_1}{\partial t} = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial x} + \nu_1\nabla^2 u_1 \quad \cdots\cdots (1)$$

$$\frac{\partial w_1}{\partial t} = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial z} + \nu_1\nabla^2 w_1 \quad \cdots\cdots (2)$$

where $u_1$ and $w_1$ denote the horizontal and vertial orbital velocities in water respectively; $p_1$ is the dynamic water pressure; $\rho_1$ and $\nu_1$ denote the density and viscosity of water respectively. The continuity equation of motion is

$$\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} = 0 \quad \cdots\cdots (3)$$

The gonverning equation for the pore-water flow is given by

$$\frac{k^*}{\gamma}\nabla^2 p_2 = \frac{n^*}{K^*}\frac{\partial p_2}{\partial t} + \frac{\partial \varepsilon}{\partial t} \quad \cdots\cdots (4)$$

where $k^*$ is the permeability coefficient of the soil bed; $p_2$ is the excessive pore-water pressure; $n^*$ is the porosity; $\gamma$ is the unit weight of pore-water; $K^*$ is the apparent bulk modulus of elasticity of pore-water; $\varepsilon$ is the volume strain of the soil particles and t is the time.

The soil is assumed to be an isotropic linear viscoelastic material and has the property of Voigt solid The dynamic equation for Voigt body is given by

$$\mu\nabla^2 u_2 + (\lambda+\mu)\frac{\partial \varepsilon}{\partial x} + \mu^*\nabla^2\frac{\partial u_2}{\partial t} + \mu^*\frac{\partial^2 \varepsilon}{\partial t\,\partial x} = \frac{\partial p_2}{\partial x} \quad (5)$$

$$\mu\nabla^2 w_2 + (\lambda+\mu)\frac{\partial \varepsilon}{\partial z} + \mu^*\nabla^2\frac{\partial w_2}{\partial t} + \mu^*\frac{\partial^2 \varepsilon}{\partial t\,\partial z} = \frac{\partial p_2}{\partial z} \quad (6)$$

where $\lambda$ and $\mu$ are elasticity constants, $\mu^*$ is the viscosity of soil.

### 2.2 Boundary conditions

1) Free surface boundary conditions

The kinetic boundary condition and the stress boundary conditions on the surface of water are given as

$$\frac{\partial \eta}{\partial t} = w_1\Big|_{z=\eta+d} \quad \cdots\cdots (7)$$

$$\rho_1\nu_1\left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x}\right)\Big|_{z=\eta+d} = 0 \quad \cdots\cdots (8)$$

$$\left(p_1 - 2\rho_1\nu_1\frac{\partial w_1}{\partial z} - \rho_1 g z\right)\Big|_{z=\eta+d} = 0 \quad \cdots\cdots (9)$$

2) Water-soil interface conditions

The velocities across the water-soil interface are continuous.

$$u_1\Big|_{z=0} = \frac{\partial u_2}{\partial t}\Big|_{z=0} \quad \cdots\cdots (10)$$

$$w_1\Big|_{z=0} = \frac{\partial w_2}{\partial t}\Big|_{z=0} \quad \cdots\cdots (11)$$

Upon the concept of effective stress, the vertical effective stress at the interface is zero and the shear stress is negligibly small, namely

$$\sigma_{zz}\Big|_{z=0} = 0 \quad \cdots\cdots (12)$$

$$\tau_{xz}\Big|_{z=0} = 0 \quad \cdots\cdots (13)$$

The pore-water pressure at the interface should be equal to the dynamic water wave pressure at the interface,

$$p_2\Big|_{z=0} = p_1\Big|_{z=0} \quad \cdots\cdots (14)$$

According to the previous assumption, the bottom is assumed to be a semi-infinite half space, thus $U_2$, $W_2$ and $p_2$ approach to zero at $z \to -\infty$,

$$u_2 \to 0,\ w_2 \to 0,\ p_2 \to 0, \quad \text{at} \quad z \to -\infty \quad (15)$$

## 3. DISCUSSION OF THE SOLUTION

Under below, the wave attenuation due soil motion and the soil seabed responses induced by wave are discussed in accordance with the exact solution presented here, and are also compared with the solutions for poro-elastic seabed model (Yamamoto).If the soil is completely saturated with water and the pore water is absolutely air-free, the apparent elasticity modulus of pore water K equals to the true elasticity modulus of water, $K=1.96\times10^9$ ($N/m^2$). Generally the shear modulus of soil varies from $4.8\times10^3$ ($N/m^2$) for silt and clay to $4.8\times10^8$ ($N/m^2$) for very dense sand(Yamamoto, 1978).If the relaxation time of the visco-elastic soil is of the same or higher order than the period of wave loading, the viscosity of soil will also has significant effect on the responses of soil bed. With the viscosity of soil being considered, the attenuation of surface wave can be calculated.

A propagating wave with a period of 10s in a waterdepth of 10m is considered in the following calculation. Fig. 1 illustrates the varia-

considered in the following calculation. Fig. 1 illustrates the varia-

tion of wave number and attenuation rate with the shear modulus of the soil. The distributions of the amplitude of soil displacement obtained from the present theory and that obtained from Yamamoto's theory are plotted in Fig.1 and Fig.2. It is found that displacements of visocelastic soil are smaller than those of elastic soil. The greater the viscosity of soil, the smaller the displacements of soil particles.

The distibutions of effective stresses and pore water pressure in the viscoelastic soil bed can also be calculated by the present solution.

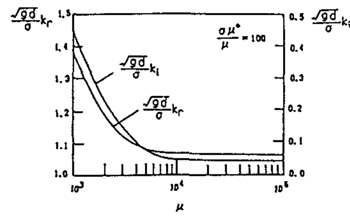


Fig.1 Variation of attenuation rate with elasticity of soil

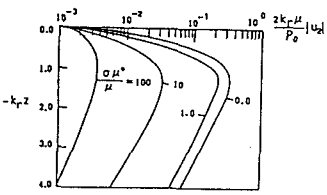


Fig.2 Vertical distribution of soil displacement

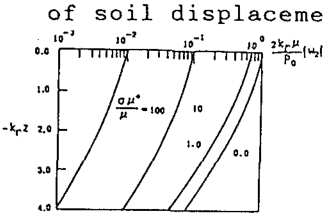


Fig.3 Vertical distribution of soil displacement

References

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