II-139 Unsteady Flushing of a Sand Bar by Overflow

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1.Introduction: When a sand bar formed at a river mouth is overflowed by a large flood , it becomes eroded due to the shear-stress of a overflow water. Here we consider an unsteady overflow erosion. Theoretical analysis was performed based on the dilatant model of the grain-inertia regime where the momentum transfers through the particle collision.

2.Experimental procedure and results:Fig.1 shows an exprimental setup. A bank made of sand whose mean diameter is 0.5mm was put inside a recirculating channel of 0.2m wide and 3m long. A thick screen was laid at an upstream part of the channel in order to reduce the wave motion of the flowing water. A video camera and a strong light were used to estimate the time development of the erosion rate.

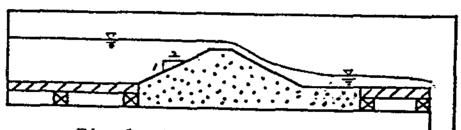

Fig.1 Experimental setup

Fig.2 and Fig.3 show the typical pattern of the overflow erosion and the time development of the erosion respectively. Here we can see three kinds of a motion,i.e.,an upstream sheet erosion, an vortex erosion and scour hole, and a downstream deposition whose physical phenomenon was already expressed[1].

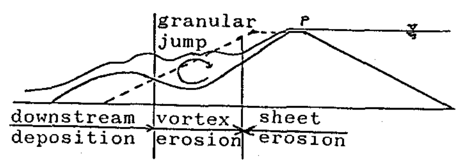

Fig.2 Typical pattern of overflow erosion

3.Theoretical analysis: Until the erosion proceeds to the upstream crown point P(of Fig.2), a crown height does not change along the time. In this case the overflow constitutes the steady flow if the inflow Qin is const.. Theory for this steady flow was already shown in the former paper[1]. After the erosion reaches that point,the crown height decreases and the flow becomes unsteady even if the inflow is constant. In this study we consider the overflow erosion for the unsteady flow.

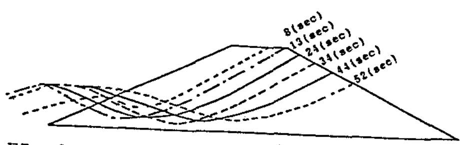

FIg.3 Time development of overflow erosion

Momentum eq. of a sand-water mixture

$$\int_{y1}^{y2} \frac{\partial(\rho u)}{\partial t} dy = T_{xy_2} - T_{xy_1} + g\sin\theta \int_{y1}^{y2} \rho \, dy \quad (1)$$

$$0 = T_{yy_2} - T_{yy_1} + g\cos\theta \int_{y1}^{y2} (\rho - \rho_f) dy \quad (2)$$

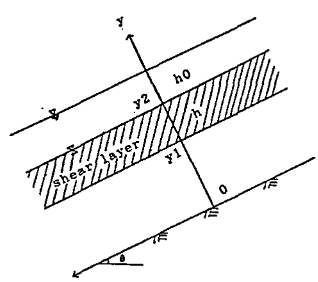

Fig.4 Two-dim. shear flow

Leibnitz's rule

$$\int_{y1}^{y2} \frac{\partial(\rho u)}{\partial t} dy = \frac{\partial}{\partial t}\int_{y1}^{y2} (\rho u) dy - \frac{\partial y2}{\partial t} \rho_{y_2} U_{y_2} + \frac{\partial y1}{\partial t} \rho_{y_1} U_{y_1} \quad (3)$$

Dynamic Coulomb criterion

$T_{xy1} = T_{yy1} * \tan\theta$

and,

$$\int_{y1}^{y2} \rho\, dy = \int_{y1}^{y2} \{\rho_s N + \rho_f (1-N)\} dy \cong (\rho_s - \rho_f) l_1 h + \rho_f h = l_2 h \quad (4)$$

where,

$y1$; bottom surface of shear layer

$y2$; top surface of shear layer

T_{xy1}, T_{xy2}; granular shear stress at $y1$ and $y2$ respectively

T_{yy1}, T_{yy2}; granular normal stress at $y1$ and $y2$ respectively

$l_2 = (\rho_s - \rho_f) l_1 + \rho_f$

$\tan\theta$; resistance coeff.

Using boundary condition $U_{y_1} = 0$, $T_{xy_2} = \rho_f g h_o \sin\theta$, and using the results of the former paper[1),

$$U_{y2} = (2/3) k_4^{1/2} h^{3/2} \sum_{j=0}^{4} B_j \quad \ldots\ldots (5)$$

$$\int_{y1}^{y2} \frac{\partial(\rho u)}{\partial t} dy = \frac{\partial}{\partial t}(U * l_2 * h) - \frac{\partial y2}{\partial t} \rho_{y2} - u_{y2} \quad (6)$$

$h_o = mh \quad (7)$, $U = l_5\, h^{3/2} \quad (8)$

where ;

$m = -\{(\Delta\rho l_1 + \rho_f)\tan\theta - \rho_f l_1 \tan\theta_D\} / (\rho_f \tan\theta)$

$l_5 = (2/3) k_4^{1/2} \sum_{j=0}^{4} \{B_j (1 - (1/(3j+2.5)))\}$

$k_4 = (l_1 \Delta\rho + \rho_f) g \sin\theta / \beta_1$

$\rho_{y2} = \Delta\rho N_o - \rho_f$

$B_j = {}_4C_j (-k_3)^j / (2j+1)$

$k_3 = (N_b - N_o) / (N_\infty - N_o)$

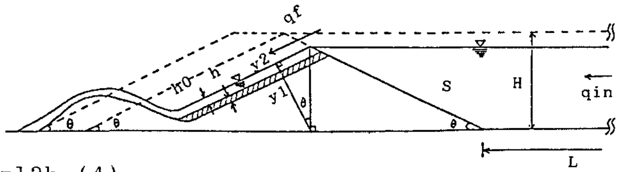

Fig.5 Definition sketch for storage eqution

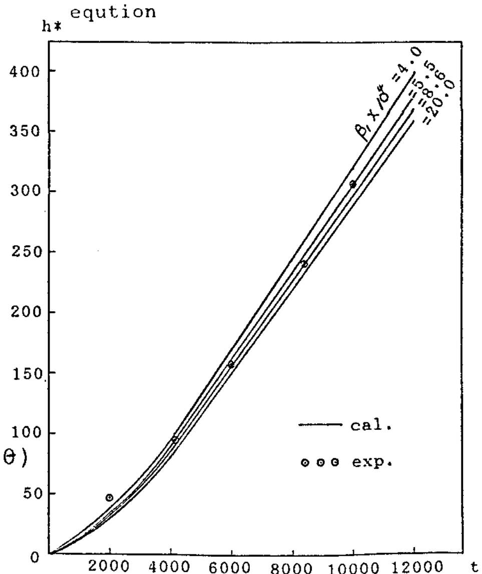

Fig.6 Non-dim. form of time development of erosion rate

Next we must consider a storage equation (Fig.5)

$$q_{in} - q_f = \partial S / \partial t \ , \quad S = (2L + H/\tan\theta) H/2 = LH \quad \text{if } L \gg H$$
$$= L(y2 + h_o)/\cos\theta$$

$$q_f = q_{in} - L/\cos\theta * \partial(y2 + mh)/\partial t \quad \ldots\ldots\ldots\ldots (9)$$

where, H; storage height S; storage q_{in}; inflow q_f; outflow l; channel length

Using the above eqs., a following resulting eq. can be obtained.

$$\partial h/\partial t + l_7\, h^{5/2} - l_8\, h^{-1/2} = l_9 \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

where, $l_7 = l_6 (\cos\theta / L)/l_o$, $l_o = 5 * l_2 * l_5 / (2 * l_3) + m$, $l_3 = 2/3 * \rho_{y2} * k_4^{1/2} \sum_{j=0}^{4} B_j$

$l_8 = l_4 / (l_3 * l_o)$, $l_4 = \rho_f m g \sin\theta + l_2 g \sin\theta - (\rho_s - \rho_f) g l_1 \cos\theta \tan\theta_d$

$l_6 = (2/3) * k_4^{1/2} [\{(-\Delta\rho l_1 - \rho_f) + \Delta\rho l_1 \tan\theta_d / \tan\theta\} \sum_{j=0}^{4} B_j$
$+ \rho_f \sum_{j=0}^{4} \{(1 - \bar{N} + (N_o - 1)/(3j+2.5) + (N_b - N_o)/(3j+5.5)\}$

$l_9 = q_{in} * (\cos\theta / L)/l_o$

Fig.6 shows the comparison between the calculational results of eq.(10) and the experimental results for some different values of a granular viscosity β_1 . Here we can see the fairly good agreement in case $\beta_1 * 10^4 = 5.5 \sim 8.6$.

4. Conclusion : Theory for the unsteady flushing motion of a sand bar based on a grain-inertia regime was proposed and a comparison between a calculational and an experimental result showed a fairly good agreement.

5. Reference :

(1) Kim, J. H., Tamai, N., and Asaeda, T. (1989), Bank erosion due to overflow. Proceeding 44th anual meeting, JSCE.