

## I-528

ESTIMATION OF FREQUENCY RESPONSE FUNCTION FROM EARTHQUAKE  
GROUND MOTIONS

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**INTRODUCTION:** The frequency response function is often used in the investigation of propagation properties of seismic waves<sup>1)</sup> and in the identification of dynamic parameters of soils<sup>2)</sup>. Because of the limitation of the number of earthquake ground motions, it is usually estimated from one event using the smoothing procedure. In this paper, the frequency response function is calculated from individual events and from an ensemble average. These results are compared and new methods are proposed for the estimation of the amplitude of the frequency response function from one event.

**PROBLEMS:** The frequency response function for the Chiba array site is estimated based on the Chiba array database<sup>3)</sup>. The ensemble frequency response functions and Fourier spectrum ratios are calculated from 14 events as shown in Fig. 1. The frequency response function is also calculated from each individual event by applying the Parzen window. Figure 2 shows an example of them for the 1985 Ibarakiken-Nanbu earthquake and this event is employed throughout the paper. The frequency response functions from the ensemble average are very close for the two horizontal components, but those obtained by smoothing exhibit considerable difference between the two components for some events. The consistency of ensemble frequency response functions for the two horizontal components indicates that the ground motion had no fixed directivity. The different frequency response functions estimated from the two horizontal components may lead to confusion in some cases.

The amplitudes of estimated frequency response functions often show a very low value at the natural frequencies. The input and output time histories are filtered with a moving frequency window (bandwidth = 0.5Hz). Figure 3 shows the filtered waves at GL-1m and GL-20m for the frequency 3.75Hz - 4.25Hz (the first natural frequency is 4.0Hz). It is found that the site strongly amplified the motions around the natural frequencies (Fig. 3) and also from the ensemble Fourier spectrum ratios shown in the Fig. 1. The low value of the frequency response function may be considered mainly caused in the stage of smoothing or averaging of the complex numbers.

**METHOD FOR ESTIMATING THE FREQUENCY RESPONSE FUNCTION WITHOUT DIRECTIVITY:**

Representing input ground motions of  $x_{SN}(t)$  and  $x_{FW}(t)$  as the complex form:  $x(t) = x_{SN}(t) + ix_{FW}(t)$ , leads to the output being the same form:  $y(t) = y_{SN}(t) + iy_{FW}(t)$ . Applying the Fourier transform to input and output yields  $Y(f) = H'(f)X(f)$ . By multiplying it by the complex conjugate of  $X(f)$  and taking the expected value, the relationship  $S'_{xy}(f) = H'(f)S'_{xx}(f)$  is obtained. Thus, the frequency response function is calculated by  $H'(f) = S'_{xy}(f)/S'_{xx}(f)$ .....(1). The frequency response function estimated by this equation is shown in Fig. 4. It is closer to that of the ensemble average than that of each individual component.

**METHOD FOR ELIMINATING THE EFFECT OF PHASE DELAY FROM SMOOTHING:**

When the frequency response function is estimated by one event, a smoothing technique is usually employed. The complex value  $S_{xy}(f)$  is obtained by smoothing its real part and imaginary part, respectively. Thus the result of the smoothing is determined not only by the amplitude but also the phase angle of each complex number. On the other hand, the phase difference between the input and output is composed of the phase delay caused by the dynamic system and various random phase angle. That is, the  $S_{xy}(f)$  can be written as  $S_{xy}(f) = A_{xy}(f)e^{i[\theta_s(f) + \theta_r(f)]}$ , in which  $A_{xy}(f)$  is the amplitude of  $S_{xy}(f)$  and  $\theta_s(f)$  and  $\theta_r(f)$  represent the phase delay from the dynamic system and the random phase angle, respectively. It has been demonstrated that the application of smoothing resulted in the reduction of the frequency response function around the natural frequencies only due to the rapid change of the phase delay<sup>4)</sup>. In order to eliminate the effect of the phase delay in the smoothing procedure, it is suggested that the amplitude of the frequency response function is calculated after the phase delay is eliminated from  $S_{xy}(f)$ , so that only random phase angles in  $S_{xy}(f)$  are dealt with in the smoothing. Then  $||h^*(f)||^2 =$

$|S_{xy}^*(f)|/S_{xx}(f)$ .....(2), in which  $S_{xy}^*(f) = S_{xy}(f)/e^{i\theta_s(f)}$  is used instead of the ordinary  $S_{xy}(f)$ . The frequency response function estimated after the elimination of phase delay is shown in Fig. 5, from which it is clear that only the sharp drop vanished comparing with that of the standard method. It is also different from the Fourier spectrum ratios where both  $\theta_s(f)$  and  $\theta_r(f)$  are neglected.

Lastly, the frequency response function is calculated by applying both methods suggested herein at the same time as shown in Fig. 6. It is very close to that of ensemble average except for near the natural frequencies. The values at natural frequencies of the Fig. 6 can be considered physically meaningful and close to the actual amplification characteristics of the site.

**REFERENCES:** 1) M. Izumi et al.: On the Coherency and the Characteristics of Transfer Functions between Seismic Waves with Instrument Array, Journal of Structural and Construction Engineering, AIJ, No. 395, pp. 131-137, 1989 (In Japanese).  
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3) F. Yamazaki, S. Nagata and T. Katayama: Construction of Strong Motion Database for the Chiba Seismometer Array, Seisan-Kenkyu, Vol. 42, No. 3, 1990 (in Japanese).  
4) L. Lu, F. Yamazaki and T. Katayama: Soil Amplification Based on the Chiba Array Database, Submitted to 8th JEES, 1990.

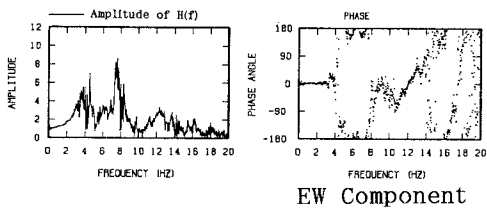


Fig. 1 Frequency Response Functions and Fourier Spectrum Ratios from ensemble average

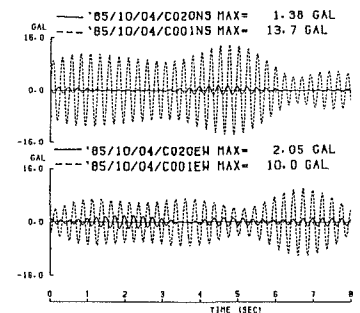


Fig. 3 Filtered Waveforms of Ground Motions at GL-1m and GL-20m

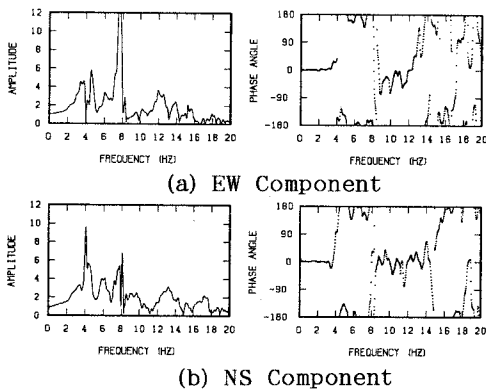


Fig. 2 Frequency Response Functions Estimated from the 1985 Ibarakiken-Nanbu earthquake

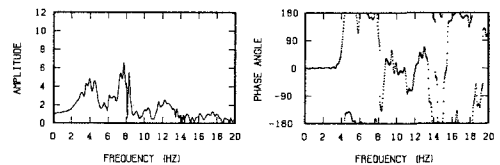


Fig. 4 Frequency Response Function from Equation (1)

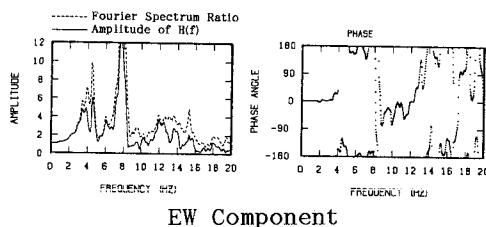


Fig. 5 Frequency Response Functions from Equation (2)

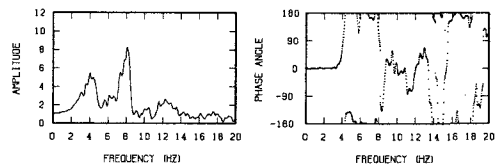


Fig. 6 Frequency Response Function Estimated by the Combination of Eqs. (1) and (2)