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A UNIVERSAL CURVE FOR MODAL DAMPING IN CABLES WITH DASHPOT

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INTRODUCTION: Viscous dashpots are often used to suppress cable vibration in cable-stayed bridges. This paper studies the modal damping of cables in cable-stayed bridges attached with a viscous dashpot. The analysis requires to solve a large complex eigenvalue problem which, inspite of advanced computer technology remains cumbersome and time-consuming. Hence, an attempt is being made to reduce computational time and cost by proposing a "universal" empirical curve in graphical form to serve as a quick design aid for a dashpot size and its location.

ANALYSIS WITHOUT SAG: The single cable is treated as a linear taut string with fixed ends (Fig.1). The equation of motion in non-dimensional form obtained after introducing the generalized coordinate using undamped modal coordinate is as under:

$$[M]\{\ddot{B}\}+[C]\{\dot{B}\}+[K]\{B\}=\{0\} \quad ....(1)$$

where $\{B\}=\left\{\frac{\tilde{B}}{L}\right\}$; $\{\tilde{B}\}^T=\{\tilde{B_1}, \tilde{B_2}, ...\tilde{B_n}\}$ is the amplitude of the generalized modal coordinate and the differentiation is with respect to τ, where $\tau = W_{01}t$; t is time.

$$[M_{ij}]=\delta_{ij} \;\; ; [K_{ij}]=i^2\,\delta_{ij} \;\; ; [C_{ij}]=\left(\frac{2C}{mW_{01}L}\right)\sin\left(\frac{\pi\, i x_c}{L}\right)\sin\left(\frac{\pi\, j x_c}{L}\right) \qquad (2), (3), (4)$$

The definition of various terms are as under:

- i = 1,2,............,n
- j = 1,2,............,n
- C : Dashpot Coefficient (N/m/sec)
- L : Length of cable (m)
- m : mass per unit length of cable (Kg/m)
- x_c : Location of viscous dashpot (m)
- T : Tension in cable (N)
- n : Number of DOF
- δ : Kronecker Delta
- $W_{01}=\frac{\pi}{L}\sqrt{\frac{T}{m}}$; first undamped natural circular frequency of taut string

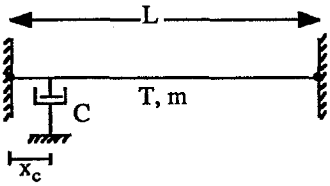

Fig.1 Cable-model

Note that the mass and stiffness matrices are diagonal but the damping matrix is non-diagonal, i.e.,. the damping is non-proportional. Uncoupling Eq.(1) by the method suggested by Foss, Ref[1] and transforming it into the eigenvalue problem, complex eigenvalues and complex eigenvectors are obtained by standard subroutine.

PRACTICAL UNIVERSAL CURVE: The equation of motion has three non-dimensional parameters: namely, $\left(\frac{2C}{mW_{01}L}\right)$, $\left(\frac{x_c}{L}\right)$ and mode number J. Treating $\left(\frac{2C}{mW_{01}L}\right)$ as non-dimensional dashpot coefficient, grouping J with it, for given $\left(\frac{x_c}{L}\right)$, one single curve covering all the six lowest modes could be obtained for modal damping ratio (MDR). As an example, modes 1 and 6 for $\left(\frac{x_c}{L}\right)$ =0.02 are shown in Fig.2. However, large number of degrees of freedom have to be considered to obtain accurate values. Again, for the same value of $\left(\frac{x_c}{L}\right)$, the mode 1 for degrees of freedom equal to 20 and 100 have been shown in Fig.3.

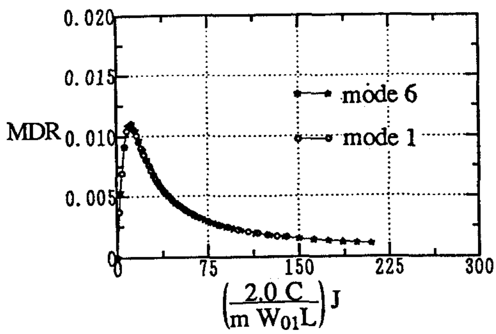

Fig.2 Single curve for modes 1 to 6 for $\frac{x_c}{L}$ =0.02

Adopting the same approach, different cases of $\left(\frac{x_c}{L}\right)$ = 0.01, 0.02, 0.03, 0.04, 0.05, 0.10 were calculated. It was observed that if the parameters were regrouped as $\left\{\left(\frac{2C}{mW_{01}L}\right) J \left(\frac{x_c}{L}\right)\right\}$ and $\left(\frac{MDR}{\left(\frac{x_c}{L}\right)}\right)$, even for these different values of $\left(\frac{x_c}{L}\right)$, again all curves could coincide and a single curve was achieved. Thus, identification of the parameter $\left\{\left(\frac{2C}{mW_{01}L}\right) J \left(\frac{x_c}{L}\right)\right\}$ was the main cause in achieving the universal curve.

Fig.4 shows the universal curve. The curve relates modal damping ratio (MDR) with dashpot coefficient (C). The curve is true for any mode number and (x_c/L) ranging up to 0.10, which has been identified as the practical limit of dashpot installation.

If the dashpot is installed at one of the nodes of a mode, the additional damping acquired for that mode would be zero. It can be noted in Eq.(4) also that if either of i or j is equal to ($1/(x_c/L)$) or the argument of sine is $n\pi$ the equation of motion (1) gets uncoupled for that mode. That is why the factor $\left(1 - \delta\left(\frac{1}{J}\right)\left(\frac{x_c}{L}\right)\right)$ has been used, so that everytime this situation occurs, the X-axis takes on zero value for which the corresponding value on Y-axis would also be zero.

It was also observed from the results obtained by eigenvalue analysis that the damped frequency of vibration for different modes is not much different from the undamped.

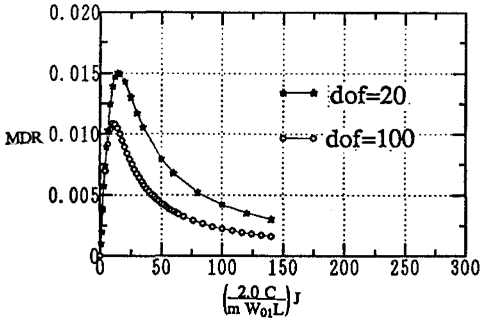

Fig.3 Effect of dof

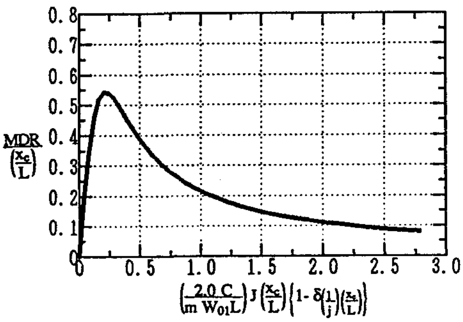

Fig.4 Universal Curve

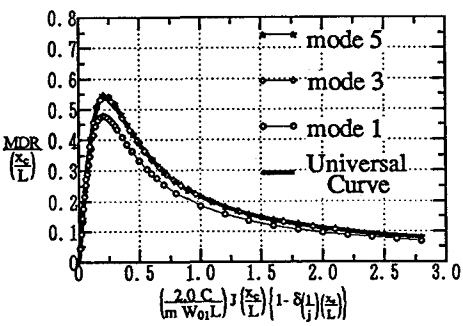

Fig.5 Effect of sag

EFFECT OF SAG: In the linearized equation of motion with cable sag, only the stiffness matrix gets altered, which becomes

$$[K_{ij}] = i^2 \delta_{ij} + \frac{2\lambda^2}{\pi^4}\left\{\frac{(1-(-1)^i)(1-(-1)^j)}{ij}\right\} \quad ..(5)$$

where $\lambda^2 = \left(\frac{m g L}{H}\right)^2 L \Big/ \left(\frac{H L_e}{EA}\right)$

Noting from Eq.(5), antisymmetric modes (J=2,4,6,.....) are not affected by sag. As mentioned in Ref.[2] , the extreme value of λ^2 for the cables in cable-stayed bridges is about 1.0. The effect of this extreme sag situation is studied for symmetric modes. Fig.5 shows that only the first (symmetric) mode is appreciably affected. The sag reduces the effectiveness of the dashpot for the first mode.

REMARKS: In cable-stayed bridges, where cables of different L,T,m exist and for each cable the value of x_c may also be different, the proposed universal curve covering all the important lower modes (not in Ref.[3]) may be helpful for design purposes. Ref.[4] also aims for the same, but the set of proposed equations has to be used once for each mode even for one value of $\left(\frac{x_c}{L}\right)$ for additional damping. The present curve is easier to use because it avoids any use of equations being graphical in form, and moreover, the accuracy has been improved.

REFERENCES: [1] Foss, K.A., Jour. Appl. Mech., 1957, Paper No. 57-A-86 [2] Gimsing, N.J., "Cable Supported Bridges , Concept and Design". [3] Kovacs, I., "Zur Frage der Seilschwingungen und der Seildampfung", Die Bautechnik,Heft 10, 1982. [4] Yoneda, M., Maeda,K, Proceedings of JSCE No.410/I-12 1989-10