I-355 IDENTIFICATION OF MDOF MODEL PARAMETERS FROM FREE-VIBRATION DATA

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BACKGROUND The numerous methods of identification of dynamic systems that have been proposed to date may be classified into three types: 1) identification of input-output relationships; 2) identification of modal characteristics; and 3) identification of model parameters [1]. The present method is of the third type, where it is assumed that an appropriate model exists whose form is known *a priori*. The structure is modelled as a multi-degree-of-freedom linear system with viscous damping: $\mathbf{M\ddot{x}+C\dot{x}+Kx=f}$ where $\mathbf{x}$ is the vector of observable degrees of freedom, and $\mathbf{M,C,K}$ are time-invariant symmetric matrices which are to be identified.

The present algorithm is meant to be as straightforward as possible, so that potential sources of errors may be easily recognized and avoided. Two or more degrees of freedom are considered, this being the first requirement in order that closely spaced modes, if any, may be identified. Data from free vibration is used ($\mathbf{f=O}$), since this type of disturbance is simpler to apply to large civil engineering structures than prescribed forces. The identification is done in the time domain, since many frequency-domain methods fail to recognize closely-spaced modes.

METHOD For convenience, the second-order differential equation of motion with n-degrees of freedom (dof) (Eq.(1)) is converted to a first-order equation with 2n-dof (Eq.(2)) by defining vector $\mathbf{y}$ (Eq.(3)) and system matrix $\mathbf{A}$ (Eq.(4)):

$$\mathbf{\ddot{x}+M^{-1}C\dot{x}+M^{-1}Kx=O} \qquad \mathbf{\dot{y}+Ay=O} \qquad \mathbf{y}=\begin{Bmatrix}\mathbf{\dot{x}}\\ \mathbf{x}\end{Bmatrix} \qquad \mathbf{A}=\begin{bmatrix}\mathbf{M^{-1}C} & \mathbf{M^{-1}K}\\ \mathbf{-I} & \mathbf{O}\end{bmatrix} \qquad (1),(2),(3),(4)$$

From measurements at discrete (digitized) values of time, the following data will be available: displacements $\mathbf{x}_i$, velocities $\mathbf{\dot{x}}_i$ and accelerations $\mathbf{\ddot{x}}_i$, where i=index of time. To reduce the amount of data that must be measured, velocities and accelerations are obtained from displacement data with the use of a differentiator filter. Band-pass filter is used to reduce noise in the original data. (Ref.[2], pp.20-24.)

The ideal goal is to find a system matrix $\mathbf{A}$ such that $\mathbf{\dot{y}}_i+\mathbf{Ay}_i=\mathbf{O}$ for every value of i, i.e., at each instant of measurement. In practice, however, the identified $\mathbf{A}$ will not exactly satisfy Eq.(2); the squared error in the equation at instant i may be denoted as $\{\mathbf{\dot{y}}_i+\mathbf{Ay}_i\}^T\{\mathbf{\dot{y}}_i+\mathbf{Ay}_i\}$. If there are N time stations, the total squared error is given by Eq.(5) below. Elements A(k,m) of system matrix $\mathbf{A}$, numbering $2n^2$ for k=1,2...,n and m=1,2,...,2n, are selected in the present method such that criterion (6) is satisfied:

$$\Pi=\sum_{i=1}^{N}\{\mathbf{\dot{y}}_i+\mathbf{Ay}_i\}^T\{\mathbf{\dot{y}}_i+\mathbf{Ay}_i\} \qquad \frac{\partial\Pi}{\partial A(k,m)}=0 \quad (k=1,2,\dots,n \quad m=1,2,\dots,2n) \qquad (5),(6)$$

If $\mathbf{z}$(k) represents the 2n elements in row k of $\mathbf{A}$, then $\mathbf{z}$(k) may be computed from a system of 2n linear algebraic equations of the following form (Ref.[2], p.26):

$$\mathbf{Bz}(k)=\mathbf{p}(k) \qquad (7)$$

where $\mathbf{B}=\mathbf{B}(x_i(k),\dot{x}_i(k),\ddot{x}_i(k);i=1,2,\dots,N;k=1,2,\dots,n)$; $\mathbf{p}_K=\mathbf{p}_K(x_i(k),\dot{x}_i(k),\ddot{x}_i(k);i=1,2,\dots,N)$.

EXAMPLES Various examples of two-degree-of-freedom system with nearly-equal frequencies and non-proportional damping were simulated in free vibration, and the respective elements of $\mathbf{M}^{-1}\mathbf{C}$ and $\mathbf{M}^{-1}\mathbf{K}$ could be identified with very accurate results (Ref.[2], pp.27-31).

To test the method on experimental data, the cable-and-girder model in Ref.[3] and Fig.1 was considered. This structure was built with nearly-equal frequencies in horizontal vibrations. In Case 1 (Fig.2), the cable and girder were initially displaced horizontally and then released. Strong beating could be observed in the time histories, suggesting a superposition of decaying sine waves with close frequencies. For comparison, Case 2 (Fig.3) was also considered where initially the girder was harmonically excited at about 9.58 Hz. Free vibration started when the exciting force was removed; less beating could be observed.

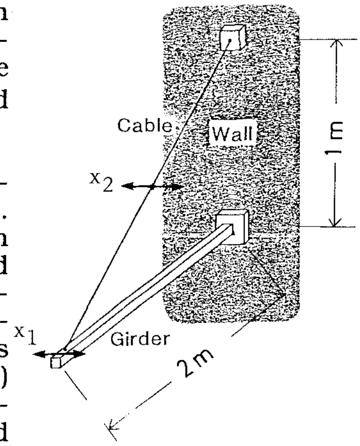

Fig.1 Cable-Girder Model

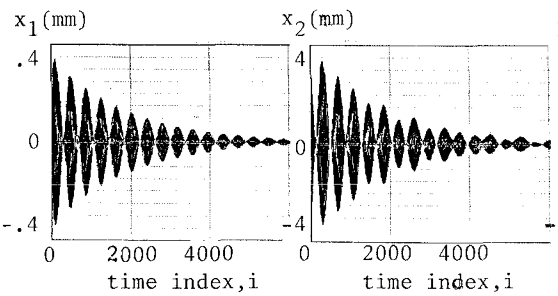

Fig.2 Case 1 (time sampling int.= 0.005s)

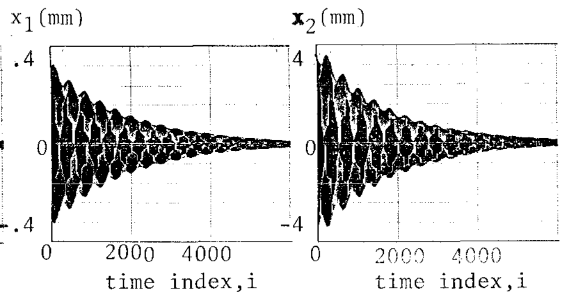

Fig.3 Case 2 (time sampling int.= 0.005s)

The identified parameters were:

$$\mathbf{M}^{-1}\mathbf{C}_{\text{Case 1}}=\begin{bmatrix} 0.2830 & -0.0070 \\ 0.7750 & 0.1378 \end{bmatrix}\text{sec}^{-1};\quad \mathbf{M}^{-1}\mathbf{K}_{\text{Case 1}}=\begin{bmatrix} 3449.18 & -17.02 \\ -2079.53 & 3426.38 \end{bmatrix}\text{sec}^{-2}$$

$$\mathbf{M}^{-1}\mathbf{C}_{\text{Case 2}}=\begin{bmatrix} 0.0772 & 0.0145 \\ -0.5230 & 0.2493 \end{bmatrix}\text{sec}^{-1};\quad \mathbf{M}^{-1}\mathbf{K}_{\text{Case 2}}=\begin{bmatrix} 3663.27 & -36.63 \\ -7516.90 & 3904.36 \end{bmatrix}\text{sec}^{-2}$$

From the system matrix, natural frequencies and modal damping ratios could be computed. Respective values may be compared for the two cases:

Case 1: f_1=9.0722Hz; f_2=9.5840Hz; ζ_1=0.00178; ζ_2=0.00184

Case 2: f_1=9.2810Hz; f_2=9.5800Hz; ζ_1=0.00064; ζ_2=0.00127

REMARKS The accuracy of the algorithm has been proved by simulated data, for systems with closely spaced frequencies. The stability can be checked by deliberately superimposing realistically random noise on the data. The total squared error, Π, may give an index of overall confidence on the parameters. However, accuracy may not be very good when one mode (mode 2 in Case 2) strongly predominates.

REFERENCES
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[2] Ohshima, H., Master's Thesis, Univ. of Tokyo, 43pp., February 1990
[3] Warnitchai, P. et al., J. Struct. Eng., JSCE, Vol.36A, pp.719-732, March 1990