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A Time-Dependent Uniaxial Modeling of Concrete as Composite Structural Material

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1. INTRODUCTION

Because seismic forces are arbitrary in their values and loading speeds, it is necessary to have a constitutive model of concrete which can give the stress for arbitrary strain-time history in finite element seismic analysis of reinforced concrete structures. The authors formerly proposed a model using a set of parallel elements to simulate the constitutive relation of concrete under any strain-time history[1], which can explain the mechanism of nonlinearity during unloading and reloading but is complicated to be applied to FEM analysis. In this paper the model is simplified by using recoverable strain during unloading and reloading.

2. MODELING TECHNIQUES

Concrete behaves as a composite material mainly during unloading and reloading. The envelope can be modeled satisfactorily with rather simple method. Here the envelope for instant loading is defined by two functions of elastic strain ε_e like in Maekawa Model[2], one is plastic strain function for instant loading ε_{p1} and the other is compressive fracture function K_e, which is approximately chosen as

$$K_e = \mathrm{Exp}(-1.6\varepsilon_n^{0.7}) \qquad (1)$$

where ε_n is a function of elastic strain. In the case of instantaneous loading, it represents the sum of elastic strain ε_e and instantaneous plastic strain ε_{p1}. The plastic function selected is

$$\varepsilon_{p1} = 10/9(-1 + (1 + 1.26\mathrm{Ln}(\varepsilon_{e1}/(\varepsilon_{e1} - \varepsilon_e)))^{0.5}) - \varepsilon_e \qquad (2)$$

in which $\varepsilon_{e1}=0.7$ is the maximum value of elastic strain. Eqs.1 and 2 give approximately the same envelope as Maekawa model.

If the loading speed is not infinite, time-dependent viscoplastic strain will develop in concrete. A strain-rate model is used for compressive viscoplastic strain

$$d\varepsilon_{vpc}/dt = E_o \cdot \gamma \cdot (\varepsilon_e - \varepsilon_y) \qquad (3)$$

$$\varepsilon_y = 0.4\mathrm{Ln}(1 + 1.2\varepsilon_{vp}) \qquad (4)$$

in which, E_o is the initial elastic model of concrete. γ is fluid parameter of concrete depending on concrete property, elastic strain and viscoplastic strain. ε_y is the yielding strain of strain-hardening slider. The plastic strain, which is affected by viscoplastic strain through compacting of concrete, is given by

$$\varepsilon_p = [\varepsilon_{p1} - \varepsilon_{vp}/(1 + 3\varepsilon_{vp})]_{max} \qquad (5)$$

in which $[\ ]_{max}$ means the maximum value ever experienced. The viscoplastic fracture function can be obtained by fitting the envelopes for different loading speeds, i.e. different viscoplastic strain.

$$K_{vp} = 0.75 + 0.25\mathrm{Exp}(-0.7\varepsilon_{vp}^{1.3}) \qquad (6)$$

The stress and strain on the envelope can be obtained as

$$\sigma = K_{vp} \cdot K_e \cdot E_o \cdot \varepsilon_e \qquad (7)$$

$$\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_{vp} \qquad (8)$$

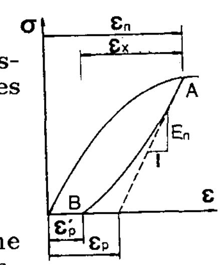

Fig. 1

In the way of unloading not only the elastic strain but also the stiffness of concrete will decrease because of the micro-cracks generated by internal tensile stresses developed due to different properties of materials in concrete. In the way of reloading there will be increasing of both elastic strain and stiffness due to internal friction and contact between the surfaces of tensile micro-cracks. The strain components on the envelope are not able to model the unloading-reloading loops properly. The recoverable strain $\varepsilon_x=\varepsilon_n-\varepsilon_p'$, which can represent the elastic properties and contact of micro-cracks is used here in the equations for unloading-reloading loops. The elastic module of unloading from point A is

$$E_n = K_{vp} \cdot K_e \cdot E_o \qquad (9)$$

The strain ε_p', which is irrecoverable when stress is zero (point B in Fig.1), and is different from the plastic strain in point A as in Ref.[1], is called irrecoverable strain. It can be obtained from Maekawa Model through ε_n

$$\varepsilon_{p1}' = \varepsilon_n - 20/7(1 - \mathrm{Exp}(-0.35\varepsilon_n)) \qquad (10)$$

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Because of the complexity of dynamic loads like earthquake forces, unloading-reloading loops, even second or higher order loops will be encountered on dynamic analysis. These loops has to be treated with thoughtfulness. Here the model is supposed to be composed of elements with unit areas. The elastic modules of these elements can be obtained by fitting a unloading curve which passes through point A,B with the tangent at A being E_n in Fig.1. The formulas have the recoverable strain ε_x as variable as shown in Fig.2. The stress of each element and the stress of the model can be obtained

$$\sigma = \Sigma(\sigma)_i \cdot (A)_i = \Sigma(E)_i \cdot (\varepsilon_e)_i \cdot (A)_i \qquad (11)$$

in which $(\ )_i$ stands for the values of i-th element. During unloading because of the internal tension generated there will be tensile viscoplastic strain developing, which depends on the unloading level and can be modeled by the following equations

$$d\varepsilon_{vpt}/dt = E_o \gamma_t(\varepsilon_{xv} - \varepsilon_{vpt}) \qquad (12)$$

$$\varepsilon_{xv} = a(1 - \varepsilon_x/(\varepsilon_x)_{max})^2 \cdot (\varepsilon_x)_{max} \qquad (13)$$

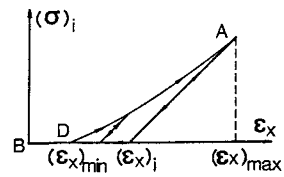

Fig. 2

in which γ_t is the fluid parameter of tensile viscoplastic strain, ε_{xv} is the maximum tensile viscoplastic strain that is possible to develop at a certain value of $(\varepsilon_x)_{max}$ and ε_x, a is a constant which can be obtained from experiments. The total strain on unloading-reloading loops is

$$\varepsilon = \varepsilon_x + \varepsilon_{pl}' + \varepsilon_{vp} + \varepsilon_{vpt} \qquad (14)$$

From above equations we can calculate stresses for any strain-time history.

3. ANALYTICAL PREDICATION

The following are some examples computed using this model. It is shown in Fig.3 that viscoplastic strain is proportional to a logarithm function of time t(day) and satisfy superposition at low stress level. Fig.4 illustrates development of viscoplastic strain under sustained loads and the creep limit line. From Fig.5 it can be seen that this model can simulate the test result under cyclic loading satisfactorily before the specimen was damaged.

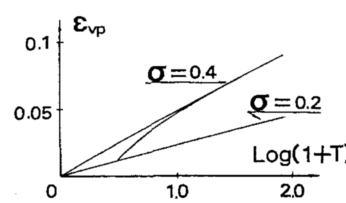

Fig.3 Viscoplastic strain

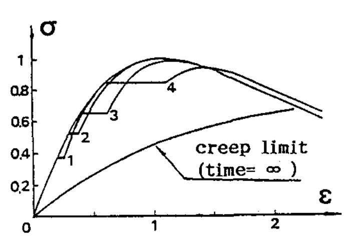

Fig.4 Constant loading

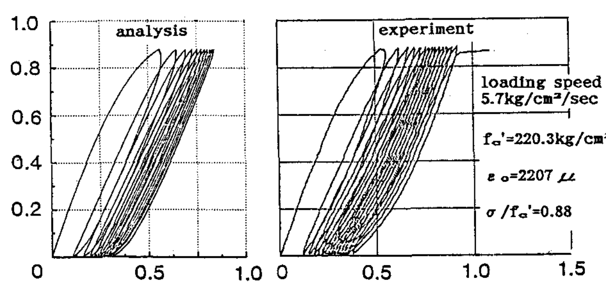

Fig.5 Response to cyclic loading

4. CONCLUSION

This model can be used for any strain-time history and is easy to be implanted in FEM. The example shows that its results agree very well with experiment results.

REFERENCES

[1]. SONG, C.M. and MAEKAWA, K., 'A Time-Dependent Uniaxial Model of Concrete Under Cyclic Loading', 2-nd Asia-Pacific Conference on Structural Engineering and Construction, 1989, pp.1510-1515.

[2]. Maekawa, K. and Okamura, H.,'The Deformational Behavior and Constitutive Equation of Concrete Using the Elasto-Plastic and Fracture Model', Journal of the Faculty of Engineering, Univ. of Tokyo, Vol.XXXVII, No.2(1983)