Ⅳ－213　四辺形三角測量に於て観測角M_iの補正値V_iを$1/\tan M_i$と角の閉合差及びλ_4で直接求める数値計算式

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§1．前　言

四辺形ＡＢＣＤ（図1.1）の観測角M_i（i=1.2.・・・8）の補正量V_i（i=1.2.・・・8）を求めるには、条件付最小自乗法により、最終的に数値の入った４元連立１次方程式の数値解を求めているのが現状である。筆者はこの４元連立１次方程式の文字解を得て、補正値V_i（i=1.2.・・・8）を$1/\tan M_i$、角の閉会差Δ秒、及びλ_4の数値を直接入力で求める数値計算式を得たので発表する。

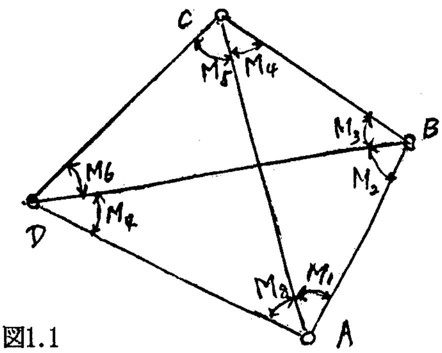

図1.1

§2．条件式ϕ_i（i = 1.2・・・5）

$$\phi_1 = (M_1+V_1) + (M_2+V_2) + (M_3+V_3) + (M_4+V_4) + (M_5+V_5) + (M_6+V_6) + (M_7+V_7) + (M_8+V_8) = 360^\circ$$

$$\phi_2 = (M_1+V_1+M_2+V_2) - (M_5+V_5+M_6+V_6) = 0$$

$$\phi_3 = (M_3+V_3+M_4+V_4) - (M_7+V_7+M_8+V_8) = 0$$

$$\phi_4 = \left[\log_e \left\{ \frac{\sin(M_4+V_4)\cdot\sin(M_6+V_6)}{\sin(M_1+V_1)\cdot\sin(M_3+V_3)} \right\} - \log_e \left\{ \frac{\sin(M_7+V_7)\cdot\sin(M_5+V_5)}{\sin(M_2+V_2)\cdot\sin(M_8+V_8)} \right\} \right] = 0$$

$$\phi_5 = f_{(v)} = V_1^2+V_2^2+V_3^2+V_4^2+V_5^2+V_6^2+V_7^2+V_8^2 = \text{minimin} = \text{最小}$$

記号：$360^\circ - \left\{ \sum_{i=1}^{i=8} M_i \right\} \equiv \Delta G$秒，$(M_5+M_6) - (M_1+M_2) \equiv \Delta C$秒，$(M_7+M_8) - (M_3+M_4) \equiv \Delta R$秒

$$\log_e \left\{ \frac{\sin M_4 \cdot \sin M_6}{\sin M_1 \cdot \sin M_3} \cdot \frac{\sin M_2 \cdot \sin M_8}{\sin M_7 \cdot \sin M_5} \right\} \equiv \Delta E$$

§3．λ_4の１元１次方程式

$$\Bigg[\frac{1}{8}\left\{ \frac{1}{\tan M_1} - \frac{1}{\tan M_2} + \frac{1}{\tan M_3} - \frac{1}{\tan M_4} + \frac{1}{\tan M_5} - \frac{1}{\tan M_6} + \frac{1}{\tan M_7} - \frac{1}{\tan M_8} \right\}^2 + \frac{1}{4}\left\{ \frac{1}{\tan M_1} - \frac{1}{\tan M_2} - \frac{1}{\tan M_5} + \frac{1}{\tan M_6} \right\}^2 + \frac{1}{4}\left\{ \frac{1}{\tan M_3} - \frac{1}{\tan M_4} - \frac{1}{\tan M_7} + \frac{1}{\tan M_8} \right\}^2 - \left\{ \sum_{i=1}^{i=8} \left(\frac{1}{\tan M_i} \right)^2 \right\} \Bigg] \times \lambda_4 \text{秒}$$

$$+ \left\{ \frac{1}{\tan M_1} - \frac{1}{\tan M_2} + \frac{1}{\tan M_3} - \frac{1}{\tan M_4} + \frac{1}{\tan M_5} - \frac{1}{\tan M_6} + \frac{1}{\tan M_7} - \frac{1}{\tan M_8} \right\} \times \frac{1}{8} \times 2\ (\Delta G)$$

$$+ \left\{ \frac{1}{\tan M_1} - \frac{1}{\tan M_2} - \frac{1}{\tan M_5} + \frac{1}{\tan M_6} \right\} \times \frac{1}{4} \times 2\ (\Delta C)\text{秒} + \left\{ \frac{1}{\tan M_3} - \frac{1}{\tan M_4} - \frac{1}{\tan M_7} + \frac{1}{\tan M_8} \right\}$$

$$\times \frac{1}{4} \times 2\ (\Delta R) = 2 \times \left\{ (\Delta E) \times \frac{1}{10^{-6} \times 4.848\ \text{radian}} \right\}$$

§4．V_i (i=1.2･･･8)の算定式

$$V_1=\frac{1}{2}\left[\left\{\frac{1}{\tan M_1}\times(-)\frac{5}{8}+\frac{1}{\tan M_2}\times(-)\frac{3}{8}+\frac{1}{8}\left(\frac{1}{\tan M_3}-\frac{1}{\tan M_4}\right)+\frac{1}{\tan M_5}\times(-)\frac{1}{8}+\frac{1}{\tan M_6}\times\frac{1}{8}+\frac{1}{8}\left(\frac{1}{\tan M_7}-\frac{1}{\tan M_8}\right)\right\}\times\lambda_4+\frac{1}{8}\times2(\Delta G)+\frac{1}{4}\times2(\Delta C)\right]$$

$$V_2=\frac{1}{2}\left[\left\{\frac{1}{\tan M_1}\times\frac{3}{8}+\frac{1}{\tan M_2}\times\frac{5}{8}+\frac{1}{8}\left(\frac{1}{\tan M_3}-\frac{1}{\tan M_4}\right)+\frac{1}{\tan M_5}\times(-)\frac{1}{8}+\frac{1}{\tan M_6}\times\frac{1}{8}+\frac{1}{8}\left(\frac{1}{\tan M_7}-\frac{1}{\tan M_8}\right)\right\}\times\lambda_4+\frac{1}{8}\times2(\Delta G)+\frac{1}{4}\times2(\Delta C)\right]$$

$$V_3=\frac{1}{2}\left[\left\{\frac{1}{8}\left(\frac{1}{\tan M_1}-\frac{1}{\tan M_2}\right)+\frac{1}{\tan M_3}\times(-)\frac{5}{8}+\frac{1}{\tan M_4}\times(-)\frac{3}{8}+\frac{1}{8}\left(\frac{1}{\tan M_5}-\frac{1}{\tan M_6}\right)+\frac{1}{\tan M_7}\times(-)\frac{1}{8}+\frac{1}{\tan M_8}\times\frac{1}{8}\right\}\times\lambda_4+\frac{1}{8}\times2(\Delta G)+\frac{1}{4}\times2(\Delta R)\right]$$

$$V_4=\frac{1}{2}\left[\left\{\frac{1}{8}\left(\frac{1}{\tan M_1}-\frac{1}{\tan M_2}\right)+\frac{1}{\tan M_3}\times\frac{3}{8}+\frac{1}{\tan M_4}\times\frac{5}{8}+\frac{1}{8}\left(\frac{1}{\tan M_5}-\frac{1}{\tan M_6}\right)+\frac{1}{\tan M_7}\times(-)\frac{1}{8}+\frac{1}{\tan M_8}\times\frac{1}{8}\right\}\times\lambda_4+\frac{1}{8}\times2(\Delta G)+\frac{1}{4}\times2(\Delta R)\right]$$

$$V_5=\frac{1}{2}\left[\left\{\left(\frac{1}{\tan M_1}-\frac{1}{\tan M_2}\right)\times(-)\frac{1}{8}+\frac{1}{8}\left(\frac{1}{\tan M_3}-\frac{1}{\tan M_4}\right)+\frac{1}{\tan M_5}\times(-)\frac{5}{8}+\frac{1}{\tan M_6}\times(-)\frac{3}{8}+\frac{1}{8}\left(\frac{1}{\tan M_7}-\frac{1}{\tan M_8}\right)\right\}\lambda_4+\frac{1}{8}\times2(\Delta G)+(-)\frac{1}{4}\times2(\Delta C)\right]$$

$$V_6=\frac{1}{2}\left[\left\{\left(\frac{1}{\tan M_1}-\frac{1}{\tan M_2}\right)\times(-)\frac{1}{8}+\frac{1}{8}\left(\frac{1}{\tan M_3}-\frac{1}{\tan M_4}\right)+\frac{1}{\tan M_5}\times\frac{3}{8}+\frac{1}{\tan M_6}\times\frac{5}{8}+\frac{1}{8}\left(\frac{1}{\tan M_7}-\frac{1}{\tan M_8}\right)\right\}\times\lambda_4+\frac{1}{8}\times2(\Delta G)+(-)\frac{1}{4}\times2(\Delta C)\right]$$

$$V_7=\frac{1}{2}\left[\left\{\frac{1}{8}\left(\frac{1}{\tan M_1}-\frac{1}{\tan M_2}\right)+\left(\frac{1}{\tan M_3}-\frac{1}{\tan M_2}\right)\times(-)\frac{1}{8}+\frac{1}{8}\left(\frac{1}{\tan M_5}-\frac{1}{\tan M_6}\right)+\frac{1}{\tan M_7}\times(-)\frac{5}{8}+\frac{1}{\tan M_8}\times(-)\frac{3}{8}\right\}\lambda_4+\frac{1}{8}\times2(\Delta G)+(-)\frac{1}{4}\times2(\Delta R)\right]$$

$$V_8=\frac{1}{2}\left[\left\{\frac{1}{8}\left(\frac{1}{\tan M_1}-\frac{1}{\tan M_2}\right)+\left(\frac{1}{\tan M_3}-\frac{1}{\tan M_4}\right)\times(-)\frac{1}{8}+\frac{1}{8}\left(\frac{1}{\tan M_5}-\frac{1}{\tan M_6}\right)+\frac{1}{\tan M_7}\times\frac{3}{8}+\frac{1}{\tan M_8}\times\frac{5}{8}\right\}\times\lambda_4+\frac{1}{8}\times2(\Delta G)+(-)\frac{1}{4}\times2(\Delta R)\right]$$

§5　計算例

M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
56° 38'~09"	11° 31'~04"	41° 41'~20"	70° 09'~48"	21° 23'~26"	46° 45'~13"	39° 56'~07"	71° 55'~43"
V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
(-)17.43秒	(-)10.83秒	2.74秒	4.5秒	0.81秒	4.95秒	(-)18.83秒	(-)16.48秒