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CONSIDERATION ON MICROSCOPIC DEFORMATION OF DILATANT GRANULAR MATERIALS

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ABSTRACT

A particle model is proposed for the derivation of a stress-dilatancy equation in a simple shear condition. On the basis of which the phenomena of initial densification and subsequent dilatancy are explained. By considering the deformation of particles arrangements, the inclination of the mean sliding plane can be related to the mean inter-particle force vector of the assemblage and a dilatancy equation is derived as a function of external stresses. If we consider the Mohr-Coulomb failure criterion can be applied to the overall flow of a granular material, a correlation between the proposed model and the dilatant double-shearing model proposed by Mehrabadi and Cowin (1978) is also derived.

1. INTRODUCTION

When a sample of a granular cohesionless material is horizontally sheared under vertical pressure, it is experimentally observed that[see, for example Taylor(1948), Rosce, Schofield, and Wroth(1958), and Rowe(1962) : (1) there is always an initial densification, the magnitude of which decreases as the initial void ratio approaches its minimum value; (2) if the sample is dense, the initial densification will be followed by dilatancy which continues until a critical void ratio is attained asymptotically; (3) if the sample is loose, i.e. the initial void ratio is larger than the critical value, then the sample densifies continuously until the critical void ratio is reached asymptotically.

Although there has been considerable theoritical work devoted to the analysis of the deformation of granular materials[see, for example, de Jong(1971), Spencer(1964, 1971), Mehrabadi and Cowin(1978)], it seems that there exists no theory which account for all the above-stated physically observed facts. It is the purpose of this paper to present a theory for two-dimensional(plane strain) deformation of granular material, which will not introduce additional kinematical or dunamical parameters than have already been presented by other researchers in the literature, nevertheless, it explains in a simple and convincing manner the phenomena of initial densification and subsequent dilatancy.

2. SIMPLE MODEL

The deformation of an assemblage of particles may be caused by: (1) the sliding and rolling between particles; (2) the compression of solid particles; and (3) the crushing of particles. According to Barden, Khayatt, and Nhightman(1969), the deformation of a specimen caused by the compression of sand particles is generally negligible. The crushing of sand particles also seems negligible if it is subjected to ordinary stress levels[Rowe(1962), Vesic and Clough(1968)]. Horne(1965) and Oda(1974) studied the mechanism of particle rolling and sliding. They concluded that the angular rotation involved in rolling may be ignored compared with the relative velocity due to sliding of two contact particles. Therefore, it is reasonable to assume that at an ordinary stress level, the deformation of an assembly of particles occurs primarily as a result of sliding only.

The inter-particle forces between two particles are shown in Fig.(1-a). The contact normal $\underset{\sim}{n}$ is defined as the vector perpendicular to the contact area of the two particles and the inter-particle force $\underset{\sim}{F}$ can be decomposed into two components, the normal force perpendicular to the contact plane, and the shear force

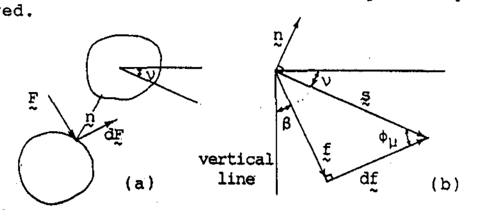

Fig. 1. Geometrical relation between inter-particle forces.

parallel to the contact plane. Due to small increment of loading(infinitisimal) applied to the soil sample, the inter-particle force changes its direction, the incremental inter-particle force df is perpendicular to F. Let $\underset{\sim}{f}$, $\underset{\sim}{n}$, $d\underset{\sim}{f}$, and $\underset{\sim}{s}$ be unit vectors representing the directions of inter-particle force, contact normal, incremental particle force, and sliding, respectively. The following condition must be satisfied for two particles at sliding, the angle between $\underset{\sim}{f}$ and $\underset{\sim}{n}$ is equal to the frictional angle ϕ_μ. It is noted that, the four vectors $\underset{\sim}{s}$, $d\underset{\sim}{f}$, $\underset{\sim}{f}$, and $\underset{\sim}{n}$ are on the same plane. The angle between $\underset{\sim}{f}$ and $\underset{\sim}{s}$ is $(\pi/2-\phi_\mu)$, $\underset{\sim}{f}$ and $\underset{\sim}{s}$ are perpendicular to $d\underset{\sim}{f}$ and $\underset{\sim}{n}$, respectively. The relationship among the unit vectors($\underset{\sim}{f}$, $d\underset{\sim}{f}$, $\underset{\sim}{s}$, and $\underset{\sim}{n}$) is shown schematically in Fig(1-b), which can be expressed as

$$\underset{\sim}{s} = \underset{\sim}{f}\sin\phi_\mu + d\underset{\sim}{f}\cos\phi_\mu \ , \quad \nu = \phi_\mu - \beta \qquad (1)$$

where β is the angle between $\underset{\sim}{f}$ and vertical line. Consider a vertical particle group with an assembly as shown in Fig.(2-a) in which the two particles Q_0 and Q, after deformation move to Q_0 and Q_1 respectively. The vector $\underset{\sim}{A}$ deforms into the vector $\underset{\sim}{A}_1$ as $\underset{\sim}{A}_1 = \underset{\sim}{A} + \delta\underset{\sim}{A}$. The strains for the simple shear condition can be defined as the relative displacement of these two particles

$$d\varepsilon_y = \frac{\delta \underset{\sim}{A}_y}{\underset{\sim}{A}} \ , \quad d\gamma = \frac{\delta \underset{\sim}{A}_x}{\underset{\sim}{A}} \qquad (2)$$

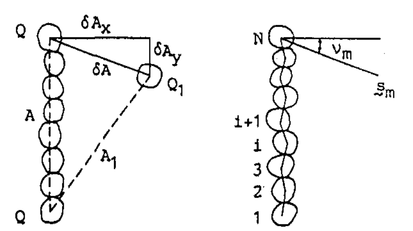

Fig. 2. Mechanism of particle deformation.

Let N and n the total number and the number of sliding particles, respectively. Then from Fig.(2-b) δA can be expressed as

$$\delta \underset{\sim}{A} = \sum_{i=1}^{n} \underset{\sim}{r}_i = \sum_{i=1}^{n} L_i \underset{\sim}{s}_i = n\, L_m \underset{\sim}{s}_m \qquad (3)$$

where r_i is the relative sliding of the upper particle to the lower particle at the i-th sliding contact with magnitude L_i and direction s_i, and L_m denotes the mean value of L_i. The mean sliding direction $\underset{\sim}{s}_m$ of all sliding contacts can be given as

$$\underset{\sim}{s}_m = \frac{1}{n} \sum_{1}^{n} \underset{\sim}{s}_i \qquad (4)$$

After substituting Eq.(1) into (4), and assuming that all contacts have the same sliding orientation due to shear stress change in simple shear condition, the result is

$$\underset{\sim}{s}_m = \underset{\sim}{f}_m \sin\phi_\mu + d\underset{\sim}{f}_m \cos\phi_\mu \qquad (5)$$

Similar to the relationship shown in Fig.(1-b) we can conclude from Eq.(5) that the inclination of the mean sliding plane is

$$\nu_m = \phi_\mu - \beta \qquad (6)$$

where β is the angle between the mean inter-particle force and the vertical line.

To relate the mean inter-particle force to the stresses in simple shear condition, consider a cubical free body with an unit area on the top and bottom as shown in Fig.(3) If we assume the angle β is given by

$$\tan\beta = \tau/\sigma \qquad (7)$$

and since $\underset{\sim}{f}_m$ by definition, has the same direction β as the resultant force $\Sigma\underset{\sim}{f}$, therefore, the angle β represents the angle between the mean inter-particle force and the vertical line. From Eq.(2) and Fig.(2), the dilatancy

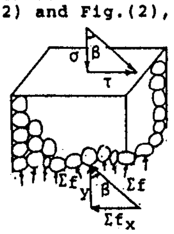

Fig. 3. Force equilibrium of a cubical element.

ratio can be expressed as the displacement ratio of the particles, and subsequently expressed as the inclination of the mean sliding plane

$$\frac{d\varepsilon_y}{d\gamma} = \frac{\delta A_y}{\delta A_x} = \tan\nu_m \qquad (8)$$

Substituting Eq.(6) into Eq.(8), and on using Eq.(7), the dilatancy equation becomes

$$\frac{d\varepsilon_y}{d\gamma} = \frac{\tan\phi_\mu - \tau/\sigma}{1 + \tan\phi_\mu \cdot \tau/\sigma} \qquad (9)$$

Eq.(9) shows that, when $\beta<\phi_\mu$, cotraction occurs ($\nu_m>0$), when β is equal to ϕ_μ, the soil has zero volume change due to shear ($\nu_m=0$), and when β is greater than ϕ_μ, dilation occurs ($\nu_m<0$).

From Fig.(1-b), we can express $\underset{\sim}{s}$ and $\underset{\sim}{n}$ as

$$\underset{\sim}{s} = \underset{\sim}{F}_y \sin\nu + \underset{\sim}{F}_x \cos\nu \;,\; \underset{\sim}{n} = \underset{\sim}{F}_y \cos\nu - \underset{\sim}{F}_x \sin\nu \qquad (10)$$

where F_x, F_y are the componets of $\underset{\sim}{F}$ in the x, y directions, respectively. The energy dissipation can be given as

$$\Delta W = \underset{\sim}{s}\ \delta\underset{\sim}{A} = \frac{F_x \tan\phi_\mu(\beta+\nu)}{\sin\phi\ \sin\nu} \delta A_y \qquad (11)$$

If we express $\Delta w = \Delta W/v$ as the incremental work per unit volume, then the total rate of work $\dot{w}$ ($\Delta w/\Delta t$) which consists of the distortional rate of work $\dot{w}$ and the dilatational rate of work $\dot{w}$, can be given as $\dot{w}^{*} = \dot{w} - \dot{w}^{*}$.

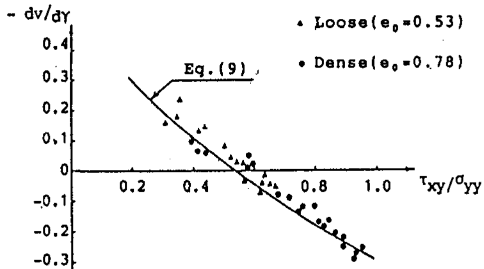

Fig. 4. Stress-dilatancy relation (after Stroud, 1971).

If we consider, the failure where the Mohr - Coulomb failure criterion can be applied to the overall flow of the granular material (in this case $\beta=\phi$) where ϕ is the of internal friction of the material, then, from Eq.(11), the distortional rate of work $\dot{w}^{*}$ can be

$$\dot{w}^{*} = \frac{\cos(\phi+\nu)}{\cos\phi\ \sin\nu} D_{kk} \qquad (12)$$

where $\dot{v}/v = trD = D_{ii}$, D is the deformation rate tensor, and τ is the maximum shear stress. Eq. (12) was first derived by Mehrabadi-Cowin [1978, Eq.(3.14)] who have used a completely different approach.

3. COMPARISON OF THEORY WITH EXISTING DATA

The stress-dilatancy relation of Eq.(9) is compared in Fig.(4) with the results of the simple shear on Lighton Buzzard sand which were conducted under drained condition by Stroud(1971). τ_{xy} and σ_{yy} in Fig.(4) indicate the shear stress and the normal stress on a horizontal plane in the centre third of a sample, respectively. The frictional angle ϕ_μ is supposed to be 27.8°. The proposed stress-dilatancy relation is noted as being in comparatively good agreement with the experimental results.

4. CONCLUSIONS

It has been shown that the angle between the mean sliding vector and the mean inter-particle force is ($\pi/2-\phi_\mu$) by considering the deformation of a particle group based on particle sliding mechanism. The dilatancy Eq. (9) based on this methodology took an entirely different approach from that of Tokue (1979), Nemat Nasser(1980), and Moroto(1987), the form of the dilatancy equation turns out to be the same. This approach may provide an alternative way towards the objective of better understanding the mechanical behavior of the granular material.

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