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WAVE PROPAGATION IN SATURATED POROUS MEDIA

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1. INTRODUCTION

In this paper. The equations of motion for a saturated porous medium , which were proposed by Biot (1956) intuitively, are derived using the principles of mixture theory. Then, an example, solved using a finite element scheme developed in the course of this study is presented.

2. DERIVATIONS OF EQUATIONS OF MOTION FOR SATURATED POROUS MEDIA, BASED ON MIXTURE THEORY

On the basis of the principles of mixture theory equations governing the behaviour of the coupled fluid-saturated porous solid system will be derived. The superscripts (s) and (f) will be employed referring to solid and fluid phases respectively, which correspond to $\alpha = 1$ and $\alpha = 2$ in the general mixture theorem expressions.

2.1 Definitions

The total mass density of the bulk material in terms of the partial mass densities of solid and fluid will be

$$\rho = \rho^{(s)} + \rho^{(f)} \tag{1}$$

Denoting the material porosity by n_f, the partial densities of solid and fluid in terms of material mass densities ρ^s and ρ^f are expressed as

$$\rho^{(s)} = (1-n_f)\rho^s \qquad \rho^{(f)} = n_f\rho^f \tag{2}$$

The total strees τ acting on the continuum region under consideration can be regarded as the summation of the partial stresses

$$\tau = \tau^{(s)} + \tau^{(f)} \quad \text{or} \quad \tau = (1-n_f)\bar{\tau}^{(s)} + n_f\bar{\tau}^{(f)} \tag{3}$$

Denoting the partial effective stress on the solid skeleton as $\bar{\tau}_{ij}^{(s)} = \tau'_{ij}$ and also the effective partial stress on the fluid as $\bar{\tau}_{ij}^{(f)} = -p\delta_{ij}$ Equation 3 can be rewritten in the following form:

$$\tau_{ij} = (1-n_f)\tau'_{ij} - n_f p\delta_{ij} \quad \text{or} \quad \tau_{ij} = (1-n_f)\tau'_{ij} - p_f\delta_{ij} \tag{4}$$

where p is the total pore fluid pressure and p_f is the part of p, acting on the fluid parts of a face on any section of the bulk material.

2.2 Mass and Linear Momentum Balance and Equations of Motion

The mass of a constituent is not necessarily conserved due to the occurence of chemical reactions between the constituents. Then the balance of mass for the constituent $s^{(\alpha)}$ is postulated as

$$\frac{d}{dt}\int_R \rho^{(\alpha)} dV + \int_{\partial R} \rho^{(\alpha)} \boldsymbol{v}^{(\alpha)} \cdot \boldsymbol{n}\ dA = \int_R m^{(\alpha)} dV \tag{5}$$

where $m^{(\alpha)}$ is the density of mass production for the constituent $s^{(\alpha)}$ arising from all the other constituents. The unit outward normal to the surface ∂R is denoted by $\boldsymbol{n}$. It is assumed that, for the mixture as a whole the total mass is conserved. Then the balance of mass for the mixture can be written together with $\sum_\alpha m_\alpha = 0$ as

$$\frac{d}{dt}\int_R \sum_\alpha \rho^{(\alpha)} dV + \int_{\partial R} \sum_\alpha \rho^{(\alpha)} \boldsymbol{v}^{(\alpha)} \cdot \boldsymbol{n}\ dA = 0 \tag{6}$$

Assuming no chemical reactions occuring between the solid and the fluid, the balance of mass for the solid phase and the fluid phase are first written, and then using the Gauss' divergence theorem, the followings are obtained

$$\frac{\partial \rho^{(s)}}{\partial t} + \boldsymbol{v}^{(s)}\frac{\partial \rho^{(s)}}{\partial \boldsymbol{x}} + \rho^{(s)}\frac{\partial \boldsymbol{v}^{(s)}}{\partial \boldsymbol{x}} = 0 \qquad \frac{\partial \rho^{(f)}}{\partial t} + \boldsymbol{v}^{(f)}\frac{\partial \rho^{(f)}}{\partial \boldsymbol{x}} + \rho^{(f)}\frac{\partial \boldsymbol{v}^{(f)}}{\partial \boldsymbol{x}} = 0 \tag{7}$$

Finally, the <u>balance of mass</u> expression for the bulk material becomes

$$\frac{D^{(s)}\rho^{(s)}}{Dt} + \frac{D^{(f)}\rho^{(f)}}{Dt} + \rho^{(s)}\frac{\partial \boldsymbol{v}^{(s)}}{\partial \boldsymbol{x}} + \rho^{(f)}\frac{\partial \boldsymbol{v}^{(f)}}{\partial \boldsymbol{x}} = 0 \tag{8}$$

In the momentum balance of the constituent $s^{(\alpha)}$, the momentum supplied to $s^{(\alpha)}$ due to chemical reactions with the other constituents and the momentum transfer due to other interaction effects such as the relative motion of the constituents have to be taken into consideration. Accordingly, the equation of linear momentum balance for the constituent $s^{(\alpha)}$ can be expressed as

$$\frac{d}{dt}\int_R \rho^{(\alpha)}\boldsymbol{v}^{(\alpha)} dV + \int_{\partial R} \rho^{(\alpha)}\boldsymbol{v}^{(\alpha)}(\boldsymbol{v}^{(\alpha)}\cdot\boldsymbol{n}) dA - \int_R m_\alpha J^{(\alpha)} dV = \int_R (\rho^{(\alpha)}\boldsymbol{F}^{(\alpha)} + \boldsymbol{\psi}^{(\alpha)}) dV + \int_{\partial R} \boldsymbol{t}^{(\alpha)} dA \tag{9}$$

Considering that the overall mass is conserved for the mixture and regarding the force $\boldsymbol{p}^{(\alpha)}$ as an internal effect, and neglecting the body forces, the <u>balance of linear momentum</u> for the solid phase and the fluid phase are first obtained, and then using Gauss' divergence theorem, we obtain the following:

$$\rho^{(s)}\left(\frac{\partial \boldsymbol{v}^{(s)}}{\partial t} + \boldsymbol{v}^{(s)}\frac{\partial \boldsymbol{v}^{(s)}}{\partial \boldsymbol{x}}\right) = \boldsymbol{\psi}^{(s)} + \frac{\partial}{\partial \boldsymbol{x}}\cdot\boldsymbol{\tau}^{(s)} \quad \rho^{(f)}\left(\frac{\partial \boldsymbol{v}^{(f)}}{\partial t} + \boldsymbol{v}^{(f)}\frac{\partial \boldsymbol{v}^{(f)}}{\partial \boldsymbol{x}}\right) = \boldsymbol{\psi}^{(f)} + \frac{\partial}{\partial \boldsymbol{x}}\cdot\boldsymbol{\tau}^{(f)} \tag{10}$$

where $\boldsymbol{t}^{(s)} = \boldsymbol{\tau}^{(s)}\cdot\boldsymbol{n}$ and $\boldsymbol{t}^{(f)} = \boldsymbol{\tau}^{(f)}\cdot\boldsymbol{n}$
Finally, the <u>balance of linear momentum</u> for the bulk material takes the following form,

$$\rho^{(s)}\frac{\partial \boldsymbol{v}^{(s)}}{\partial \boldsymbol{x}} + \rho^{(f)}\frac{\partial \boldsymbol{v}^{(f)}}{\partial \boldsymbol{x}} = \frac{\partial}{\partial \boldsymbol{x}}\cdot(\boldsymbol{\tau}^{(s)} + \boldsymbol{\tau}^{(f)}) = \frac{\partial}{\partial \boldsymbol{x}}\cdot\boldsymbol{\tau} \tag{11}$$

Equation of motion for the total system can be obtained from the balance of linear momentum of the total system. Note that the diffusion force term $\psi^{(\alpha)}$ vanishes for the mixture as a whole in the linear momentum balance expression. Let us take $\boldsymbol{v}^{(s)} \equiv \boldsymbol{v}$ for simplicity, then the Lagrangian description of the motion will be :

$$\nabla \cdot \boldsymbol{\tau} = (1 - n_f)\rho^{(s)}\dot{\boldsymbol{v}} + n_f \rho^{(f)}\dot{\boldsymbol{v}} \tag{12}$$

Introducing the relative flow vector $\boldsymbol{w}$ of the fluid with respect to solid (volume per unit area) as $\boldsymbol{w} = n_f(\boldsymbol{u}^{(f)} - \boldsymbol{u}^{(s)})$ the equation of motion derived for the bulk material (12) can be rewritten in terms of $\dot{\boldsymbol{v}}$ and $\ddot{\boldsymbol{w}}$ as given below.

$$\nabla \cdot \boldsymbol{\tau} = \rho\dot{\boldsymbol{v}} + \rho^{(f)}\ddot{\boldsymbol{w}} \quad \text{or} \quad \nabla \cdot \boldsymbol{\tau} = \rho\ddot{\boldsymbol{u}} + \rho^{(f)}\ddot{\boldsymbol{w}} \tag{13}$$

Equation of motion for the pore fluid can be obtained from the balance of linear momentum for the fluid phase only. Then, using the relation (10 $)_2$, the Lagrangian description of the motion will be

$$\nabla \cdot \boldsymbol{\tau}^{(f)} = \boldsymbol{\psi}^{(f)} - n_f \rho^f \dot{\boldsymbol{v}}^{(f)} \tag{14}$$

Introducing a phase-interaction force of Zhukovski-type together with a fluid resistance of Darcy-type, and carrying out some manipulations, the equation of motion for the pore fluid can be expressed as

$$\nabla \cdot \bar{\boldsymbol{\tau}}^{(f)} = \rho^{(f)}\dot{\boldsymbol{v}}^{(f)} + \frac{\eta}{k}\dot{\boldsymbol{w}} \quad \text{or} \quad \nabla \cdot \bar{\boldsymbol{\tau}}^{(f)} = \rho^{(f)}\ddot{\boldsymbol{u}}^{(f)} + \frac{\rho^f}{n_f}\ddot{\boldsymbol{w}} + \frac{\eta}{k}\dot{\boldsymbol{w}} \tag{15}$$

As it can be noticed easily, the equations of motion we have derived for the bulk material as a whole and for the pore fluid are the same as the equations of motion proposed by Biot (1956).

3. NUMERICAL SCHEME FOR THE SOLUTIONS EQUATIONS OF MOTION

For the solution of the coupled form of the equations of motion given in Eqns 13 and 15, a finite element scheme for space-domain together with constitutive laws of the Biot-type and a finite-difference scheme is employed (See Ekşioğlu (1989) for details).

4. EXAMPLES AND CONCLUSIONS

Because of lack of space, an example of wave propagation in an half space subjected to an exponentially decaying uniform dynamic loading applied at top is only given (Fig. 1). Figure 2a,b,c show the responses of displacement, velocity and acceleration at a point which is located at the the surface where the load was applied. Figure 2d shows the response of total stress and fluid pressure at an element very close to the point of load application.

In this study, equations of motion for the binary mixture of a fully saturated porous medium have been briefly derived following the general assumptions of mixture theory. It is shown that the resulting forms of the equation are the same as those intuitively proposed by Biot (1956). An example analysed by the numerical scheme developed in the course of this study is given (more examples will be presented in the poster session).

REFERENCES

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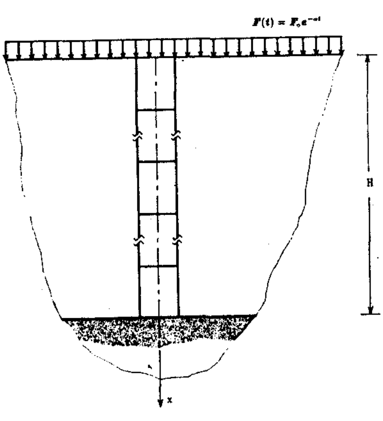

Figure 1. One dimensional finite element model of porous material column

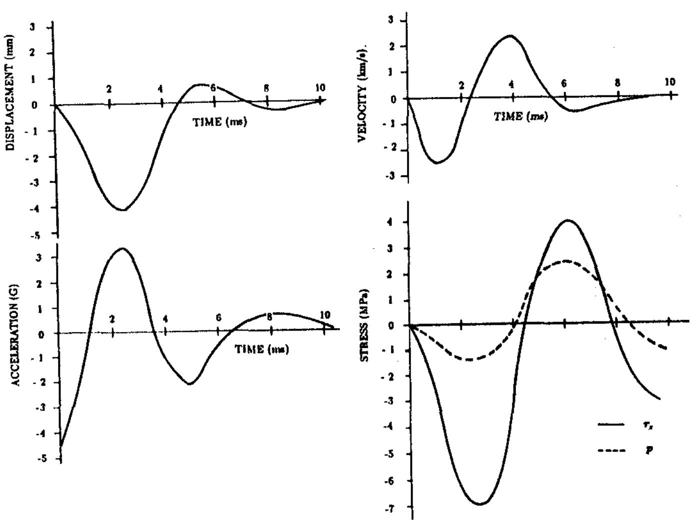

Figure 2. Calculated displacement, velocity and acceleration and total-stress and fluid pressure responses