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BANK EROSION DUE TO OVERFLOW

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1.Introduction: There are two cases on the bank erosion,i.e.,the overflow erosion and the seepage erosion. Here we consider the overflow erosion where the velocity of the overflow water is higher enough to pass ahead the seepage velocity. The experimental results of the bank erosion were observed and the theoretical analysis was performed considering the momentum eq. and the stress-strain relationships for the grain-inertia regimes.

2.Experimental Setup : Fig.1 shows the experimental arrangement.The bank made of sand(d=0.5mm)was put inside the recirculating rectangular channel of 0.2m wide and 3m long. A thick screen was laid at the upstream part of the channel in order to reduce the wave motion and the water was flowing from the upstream part. The video camera and the strong light were used to estimate the erosion rate.

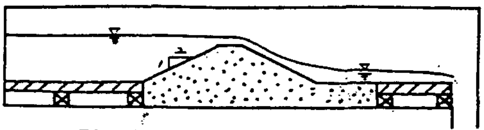

Fig.1 Experimental setup

3.Experimental Observations: Fig.2 and 3 shows the typical pattern of the erosion and the erosion rate respectively, and the following observations were obtained

1)The debris flow begins from the starting point of the granular jump and the immature debris flow occurs in front of that point.

2)The sheet erosion occurs in the area of the immature debris flow due to the shear stress and its slope was parallel to the initial bank slope,i.e.,θ=const..

3)The vortex erosion and the resulting large scour hole occurs at the downstream part of the bank due to the line vortex and this scour hole proceeds to the upstream direction finally stopped at the downstream crown point P1.

4)The bed sediments moves to the lowest part of the scour hole and suspended into the water at the right angle of the bed surface.

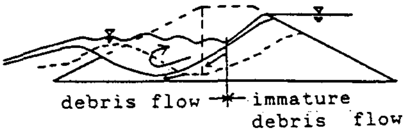

Fig.2 Erosion pattern

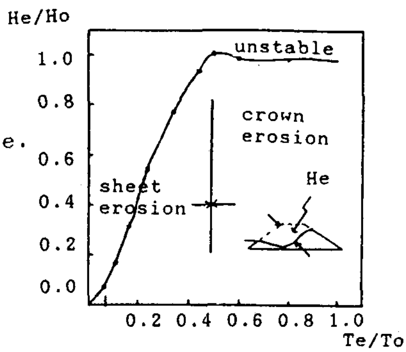

Fig.3 Erosion rate

4.Theoretical Analysis:Consider the 2-dim. sheet flow where the erosion occurs parallel to the initial bank slope at the upper part of the downstream bank slope. In this case we assume the uniform flow and there is no net flux of sands across the surface at y=0.

Continuity eq. of the mixture

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \qquad \rho=\rho_s N+\rho_f(1-N);\ \text{mixture density}$$

Momentum eq. of the mixture

$$\frac{\partial(\rho u)}{\partial t}=\frac{\partial T_{xy}}{\partial y}+\rho g\sin\theta$$

$$0=\frac{\partial T_{yy}}{\partial y}+(\rho-\rho_f)g\cos\theta$$

where, T_{xy};granular shear stress

T_{yy};granular normal stress u;mixture velocity

ρ_s;granular density ρ_f;fluid density

N;granular volume fraction

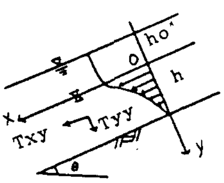

Fig.4 Two-dim. sheet flow

Until the erosion proceeds to the upstream crown point P2 of Fig.2, the crown height does not

decrease and the steady flow exists.
Letting $\int_0^y N\,dy \cong (N_o + lN_o)y = l_1 y$, and using the boundary condition $T_{xy} = -\rho_f g h_o \sin\theta$ at $y=0$, following two eqs. can be obtained.

$$-T_{xy} = \Delta\rho\, g l_1 y \sin\theta + \rho_f g(h_o + y)\sin\theta \quad (1)$$

$$-T_{yy} = \Delta\rho\, g l_1 y \cos\theta \quad (2)$$

For the rapid shear flows, the stresses are,

$$T_{xy} = -\mu_1 \left(\frac{\partial u}{\partial y}\right)^2 - \rho_f g h_o \sin\theta \quad \left(\because \frac{\partial u}{\partial y} < 0\right) \quad (3)$$

$$T_{yy} = -\alpha(b^{-1}+1)\left(\frac{\partial N}{\partial y}\right)^2 - \mu_o \left(\frac{\partial u}{\partial y}\right)^2 \quad (4)$$

where; $\mu_o = \beta_o \left(\frac{N_\infty - N_o}{N_\infty - N}\right)^\beta$, $\mu_1 = \beta_1 \left(\frac{N_\infty - N_o}{N_\infty - N}\right)^\beta$, $\beta_1/\beta_o = \tan\phi_o$
$b = k(N - N_o)$, $k, \alpha, \beta_o, \beta$:const.
N_∞;volume fraction at the densest packing
N_o;volume fraction at y=0
ϕ_D;dynamic angle of internal friction
Using the above four eqs. and the boundary condition $N=N_b$ at $y=h$, the following approximate non-dim. distribution of volume fraction can be obtained.

$$\frac{N - N_o}{N_b - N_o} = \left(\frac{y}{h}\right)^3 \quad (5)$$

where h;depth of shear layer

Fig.5 shows eq.(5). In order to determine h, we use the dynamic Coulomb criterion, $T_{xy} = T_{yy}\tan\phi_y$.
From (1)&(2),

$$h = \frac{-\rho_f \tan\theta}{(\Delta\rho l_1 + \rho_f)\tan\theta - \Delta\rho l_1 \tan\phi_y}, \quad \Delta\rho = \rho_s - \rho_f \quad (6)$$

The average volume fraction N is,

$$\bar{N} = \frac{1}{h}\int_0^h N\,dy = \frac{3N_o + N_b}{4} \quad (7)$$

For the determination of the velocity profile of the shear layer, from (1)&(3),

$$u = \frac{2}{3}K_4^{1/2} h^{3/2} \sum_{j=0}^{4} \frac{{}_4C_j(-K_3)^j}{2j+1}\left[1 - \tilde{y}^{(2j+1)}\right], \quad \tilde{y} = (y/h)^{3/2}, \quad K_3 = \frac{N_b - N_o}{N_\infty - N_o}, \quad K_4 = (l_1\Delta\rho + \rho_f)g\sin\theta/\beta_1$$

Using the boundary condition $u=u_o$ at $y=0$,

$$u_o = \frac{2}{3}K_4^{1/2} h^{3/2} \sum_{j=0}^{4} \frac{{}_4C_j(-K_3)^j}{2j+1}$$

So, the non-dim. velocity profiles are,

$$\frac{u}{u_o} = \sum_{j=0}^{4} B_j\left[1 - \tilde{y}^{(2j+1)}\right] \Big/ \sum_{j=0}^{4} B_j, \quad B_j = {}_4C_j(-K_3)^j/(2j+1) \quad (8)$$

Fig.6 shows eq. (8). The average velocity $\bar{u}$ is, $\bar{u} = 1/h \cdot \int_0^h u(y)\,dy$

$$= \frac{2}{3}K_4 h^{3/2} \sum_{j=0}^{4}\left[B_j\left(1 - \frac{1}{3j+5/2}\right)\right] \quad (9)$$

The granular discharge Q_s is,

$$Q_s = \rho_s \int_0^h N(y)u(y)\,dy = \frac{2}{3}K_4 h^{5/2} \rho_s \sum_{j=0}^{4}\left[B_j\left(\bar{N} - \frac{N_b - N_o}{3j+11/2} - \frac{N_o}{3j+5/2}\right)\right] \quad (10)$$

The fluid discharge Q_f is,

$$Q_f = -\rho_f h_o u_o + \rho_f \int_0^h [1 - N(y)]u(y)\,dy$$
$$= \left[\left\{(\Delta\rho l_1 + \rho_f) - \Delta\rho l_1 \frac{\tan\phi_y}{\tan\theta}\right\} \sum_{j=0}^{4}\left\{B_j + \rho_f\left(1 - \bar{N} + \frac{N_o - 1}{3j+5/2} + \frac{N_b - N_o}{3j+11/2}\right)\right\}\right] h^{5/2} \quad (11)$$

Fig. 7 shows eq. (11). As we can see, the differences between the experimental and the theoretical values are not so large.

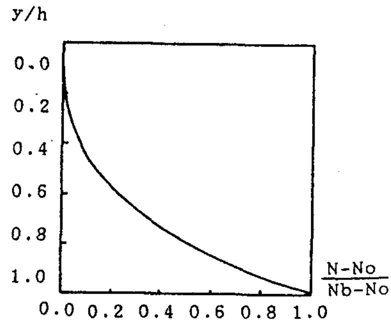

Fig.5 Non-dim. distribution of volume fraction

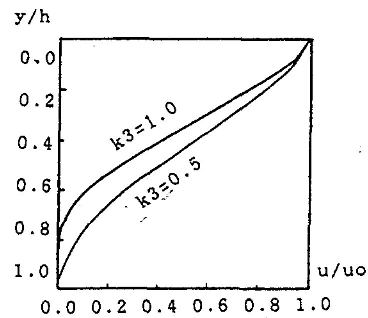

Fig.6 Non-dim. velocity profile

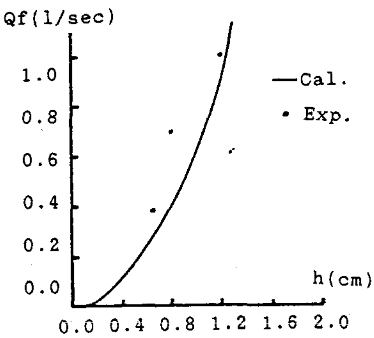

Fig.7 Relationship beween Qf and h

5.Conclusion; In this paper the theoretical analysis for the sheet erosion in case of the uniform flow was proposed. The theory for the the unsteady flow and the comparison between the experimental and theoretical results will be discussed later.

6.References;

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