I-220 Regularisation in 3D BIE for Anisotropic Elastodynamic Crack Problems

N. Nishimura, Kyoto Univ., Mem. JSCE

1. Introduction

Let Γ be a smooth piece of curved surface in R^3, having a smooth edge $\partial\Gamma$. The elastodynamic crack problem is formulated as follows: Find a displacement $u_i(\mathbf{x})$ and stress $\tau_{ij}(\mathbf{x})$ which satisfy the field equations

$$(1) \qquad \begin{cases} \tau_{ij,j} + \rho\omega^2 u_i = 0 \\ \frac{1}{2}(u_{i,j} + u_{j,i}) = D_{ijkl}\tau_{kl} \quad \text{in } R^3 \setminus \bar{\Gamma}\,, \end{cases}$$

boundary condition and regularity condition

$$\tau_{ij}^{\pm} n_j = t_i \quad \text{on } \Gamma, \qquad [u_i] = 0 \quad \text{on } \partial\Gamma$$

and the radiation condition, where D_{ijkl}, ρ, ω and t_i are the compliance tensor, density, frequency and the traction given on Γ. Also, n_i stands for the unit normal vector to Γ, superposed $+$ and $-$, respectively, indicate the limit from the side of Γ into which $\mathbf{n}$ points and the limit from the other side, $_{,i} = \partial/\partial x_i$, and $[u_i] = u_i^+ - u_i^-$.

The double layer potential approach for this problem uses an 'integral' equation

$$t_i(\mathbf{x}) = \text{p.f.} \int_\Gamma \Sigma_{ijkl}(\mathbf{x}-\mathbf{y}) n_j(\mathbf{x}) n_l(\mathbf{y}) f_k(\mathbf{y}) \mathrm{d}S_y, \quad \mathbf{x} \in \Gamma$$

where $f_i(= [u_i])$ is the unknown vector function on Γ, and Σ is a kernel function which satisfies

$$(2) \qquad \Sigma_{ikab,kj}(\mathbf{x}) + \Sigma_{jkab,ki}(\mathbf{x}) + 2\rho\omega^2 D_{ijkl}\Sigma_{klab}(\mathbf{x}) = -\rho\omega^2(\delta_{ia}\delta_{jb} + \delta_{ib}\delta_{ja})\delta(\mathbf{x})$$

with Dirac's delta $\delta(\mathbf{x})$. With $\mathbf{f}$, one computes τ_{ij} by

$$\tau_{ij}(\mathbf{x}) = \int_\Gamma \Sigma_{ijkl}(\mathbf{x}-\mathbf{y}) n_k(\mathbf{x}) f_l(\mathbf{y}) \mathrm{d}S_y$$

and u_i by using (1).

A difficulty inherent to the numerical analysis based on this approach is the strong singularity of $\Sigma(\mathbf{x})$, which is of the order of $|\mathbf{x}|^{-3}$ as $|\mathbf{x}| \to 0$. This singularity is usually removed with the help of the "regularisation" [1][2]. In [1] Nishimura & Kobayashi have shown that this regularisation is carried out in an automatic manner, once one finds a decomposition of the form

$$(3) \qquad \Sigma_{ijkl}(\mathbf{x}) = (\text{curl}\,)_i(\text{curl}\,)_j(\text{curl}\,)_k(\text{curl}\,)_l\Phi_{....}(\mathbf{x}) + \Psi_{ijkl}(\mathbf{x})$$

where Φ and Ψ are kernels which behave essentially as $O(|\mathbf{x}|)$ and $O(|\mathbf{x}|^{-1})$ as $|\mathbf{x}| \to 0$, respectively. Φ is called the stress function for Σ. We here present explicit formulae for Φ and Ψ for the general case of anisotropic elastodynamics.

2. Results

Using Voigt's notation we rewrite the Fourier transform of (2) as

$$(\mathbf{K} - \rho\omega^2\mathbf{D})\hat{\Sigma} = \rho\omega^2\mathbf{1},$$

where ^ indicates the F.T. with respect to $\mathbf{x}$ $(\mathbf{x} \to \xi)$ and $\mathbf{K}$ is the 6×6 version of the F.T. of the differential operator in (2). Obviously one has

$$\hat{\Sigma} = (\mathbf{K} - \rho\omega^2\mathbf{D})^{-1}\rho\omega^2 = \frac{\{\mathrm{cof}(\mathbf{K} - \rho\omega^2\mathbf{D})\}^T}{\det(\mathbf{K} - \rho\omega^2\mathbf{D})}\rho\omega^2. \tag{4}$$

We have the following results:

(a) The determinant in (4) can be factored out by $(\rho\omega^2)^3$.

(b) The cofactor in (4) can be factored out by $(\rho\omega^2)^2$.

(c) The F.T. of the stress function Φ in (3) is written as

$$\begin{aligned}\hat{\Phi}_{klij} &= \frac{(\delta_{ks}\delta_{lm} + \delta_{km}\delta_{ls})(\delta_{ia}\delta_{jc} + \delta_{ic}\delta_{ja})e_{tun}e_{bvd}\xi_u\xi_v D_{stab}D_{mncd}}{2[\det(\mathbf{K} - \rho\omega^2\mathbf{D})/(\rho\omega^2)^3]} \\ &= \frac{(D_{ktib}D_{lnjd} + D_{ltib}D_{knjd} + D_{ktjb}D_{lnid} + D_{ltjb}D_{knid})e_{tun}e_{bvd}\xi_u\xi_v}{2[\det(\mathbf{K} - \rho\omega^2\mathbf{D})/(\rho\omega^2)^3]}.\end{aligned} \tag{5}$$

3. Concluding Remark

It is seen that the present formulation does not destroy the causality in time domain. This is, however, not to be the case with the Nédélec formulation given in [2].

References

[1] Nishimura, N. and Kobayashi, S.(1989), A regularized boundary integral equation method for elastodynamic crack problems, to appear in Compt. Mech.

[2] Nédélec, J.C.(1983), Le potentiel de double couche pour les ondes élastiques, Internal repport of Centre de Mathématiques Appliquées, Ecole Polytechnique.