III-298 SPREADSHEET SOLUTION FOR STRESS DISTRIBUTION IN ELASTIC MEDIA WITH NON-UNIFORM ELASTIC MODULUS PROFILE

T. Adachi, M. Horita, M. Emir
Professor, Instructor, Graduate Research Student
Department of Civil Engineering, Kyoto University, Kyoto

Introduction

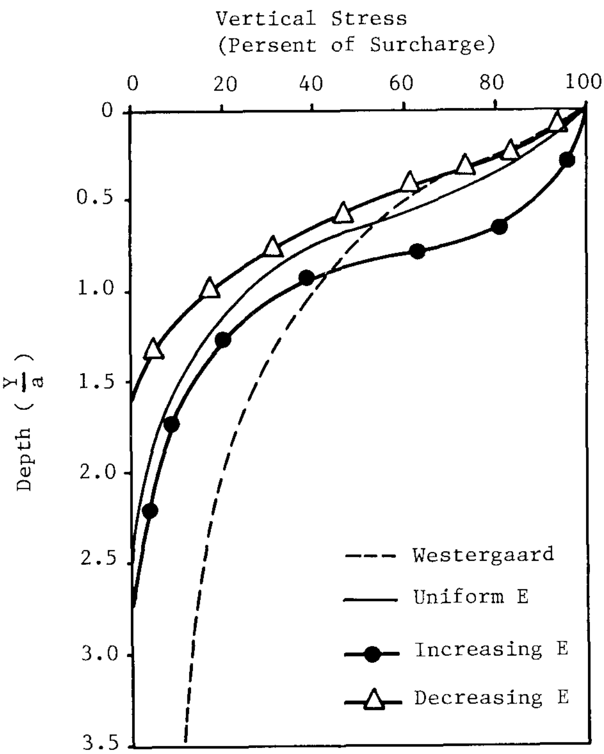
One of the important concerns in foundation engineering is the estimation of vertical stress distribution in the soil mass due to a surface load. In this case, the main interest lies in obtaining the stress profile beneath the surcharge and controlling the stress level a given soil element will experience.
The theory of elasticity is often used by foundation engineers to calculate stresses in the soil mass. Solutions to this problem have been obtained by Boussinesq (1885), Newmark (1935), Westergaard (1938). These solutions have been presented in the form of influence charts, among the more common of which are the ones proposed by Westergaard (1938), Newmark (1942), Leonards (1962), Peck et al. (1974).

Layered Elastic Systems

In the case of layered soil where differences in modulus of elasticity are marked, solutions other than the above are required. Such solutions have been obtained by Burmister (1943,1958,1967) who developed expressions and provided graphical results for stresses in two and three layered systems under a circular loaded area, Acum et al. (1951) who also provided graphical solution techniques, Harr (1966) who gave detailed expressions for the computation of stresses due to various surface loads, Poulos et al. (1974) who provided numerical as well as graphical solution charts for multilayered soil and rock masses.
It is also desirable to obtain solutions for the stress distribution in a soil or rock mass with a non-uniform elastic modulus profile as well as a combination of a layered system and non-uniform elastic modulus profiles. Such a solution can be obtained by numerically integrating the governing differential equation and in the following, this will be further explained.

Spreadsheet Solution of Stress Distribution Problem

The governing equation for the problem is obtained by combining the equilibrium, compatibility equations, Hooke's law and Airy's stress function in plane strain. The resulting equation contains the biharmonic equation as well as a term which accounts for the variation of the elastic modulus in the vertical direction. For uniform **E** this term reduces to zero.
The governing equation is solved by the finite difference method which is particularly well suited for spreadsheet applications. The use of a spreadsheet such as the Lotus 1-2-3 eliminates algorithmic programming and provides the flexibility and the spotaneity necessary for enginnering judgment. The governing equation is typed in one cell and copied into all other cells which make up the domain. The solution domain is visually created, each cell becoming a node on the finite difference grid which can be viewed on the screen and can be adapted to the geometry of the problem. This permits the analysis of irregularly shaped solution domains as easily as regularly shaped ones. The boundary conditions and the surcharge are also contained in cells, located at their "natural" locations with respect to the solution domain. The assignment of a Young's modulus for each node permits the handling of any kind of modulus distribution in the soil mass, including voids and perfectly rigid regions. These regions are also at their "natural" locations in the domain.
In the present study, solutions for a soil with a decreasing E with depth and a soil with an increasing E with depth are obtained and compared with the stress distribution in a soil with uniform E with depth (Fig. 1)



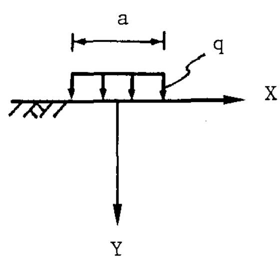


Fig.1 Vertical Stress Distribution under the Center Line of Loaded Area

References

Boussinesq,(1885) "Application des Potentiels a l'Etude de l'Equilibre et des Mouvements des Solides Elastiques",Gauthier-Villars,Paris

Newmark,(1935) "Simplified Computation of Vertical Pressures in Elastic Foundations", U. of Illinois Eng. Exp. Bull. No.367

Westergaard,(1938) "A Problem of Elasticity Suggested by a Problem in Soil Mechanics: Soft Material Reinforced by Numerous Strong Horizontal Sheets",Contributions to Mech. of Solids, S. Timoshenko 60th Anniversary Volume,MacMillan Publishing Co.,N.Y.

Newmark,(1942) "Influence Charts for Computation of Stresses in Elastic Foundations",U. of Illinois Eng. Exp. Bull., No.338

Leonards,(1962) "Engineering Properties of Soils", Foundation Engineering,Edited by G.A. Leonards,Chapter 2,McGraw-Hill,N.Y.

Peck,Hansen & Thornburn,(1974) Foundation Engineering, 2nd Edition, John Wiley & Sons, N.Y.

Burmister,(1943) "The Theory of Stress and Displacements in Layered Systems and Applications to Design of Airport Runways",Proc. Highway Research Board, Vol.23, p.127

Burmister,(1958) "Evaluation of Pavement Systems of the WASHO Road Test Layered System Methods", Highway Research Board Bull., No. 177

Burmister,(1967) "Applications of Dimensional Analysis in the Evaluation of Asphalt Pavement Performances", Proc. 5th Paving Conf., Albuquerque N.M., p.22

Acum & Fox,(1951) "Computation of Load Stresses in a 3 Layer Elastic System", Geotechnique, Vol.2, No.4

Harr,(1966) Foundations of Theoretical Soil Mechanics, McGraw-Hill, N.Y.

Poulos & Davis,(1974) Elastic Solutions for Soil and Rock Mechanics