

## II-36 FORECASTING OF TIDAL RIVER WATER SURFACE LEVELS DURING TYPHOON PERIODS

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## INTRODUCTION

This report describes techniques for the modelling and prediction of water levels when flood passes through the tidal reach of a river during typhoon periods. In these periods, Hashino and Kanda (1985) have shown that there are possibilities that storm surge running up the tidal reach of a river may occur simultaneously with the passage of flood, which should make the water surface levels in the tidal reach higher than those due to either flood or surge alone. Two simple time series models, a periodic function model and an ARMAX model, are used to describe the water-level variations at different locations in the tidal reach. The Kalman Filter is used to automatically update the estimates of model coefficients or make forecasts recursively, based on the latest information. These techniques are applied to the problem of predicting the water levels in the tidal reach of the Sendai River, Kagoshima Prefecture. In addition, evaluation of the rate at which the water surface level rises and specification of noise covariances for the implementation of the filter are discussed.

## MODELLING AND PREDICTION TECHNIQUES

The power spectra of the water level hydrographs at Kumisaki, Takaeomote, Sendai and Onobuti Gauging Stations during the typhoon in August of 1971 are obtained by maximum entropy method (MEM). The tidal reach extends up to Onobuti Gauging Station. These results of the spectral analyses are used to decide which of the two time series models is appropriate to describe the variations of water levels at certain locations in the tidal reach.

The water level time series in Kumisaki and Takaeomote at the lower end of the tidal reach may be represented adequately by a periodic function in the form:

$$y(t) = M_y(t) + \sum_{i=1}^q (A_i(t) \sin 2\pi f_i t + B_i(t) \cos 2\pi f_i t) + w(t) \quad (1)$$

where  $y(t)$  is tidal water level at time  $t$ ,  $M_y(t)$  is mean of the sequence,  $q$  is number of significant frequency components,  $f_i$  is frequency component,  $A_i(t)$  and  $B_i(t)$  are periodic coefficients, and  $w(t)$  is stochastic component which is assumed to be white Gaussian noise with zero mean and variance  $R$ . Two frequency components,  $f_1 = 1/12$  and  $f_2 = 1/6$  in cycles per hour, are identified from the results of the spectral analyses.

A simple ARMAX model is used to describe the variations in water levels in Sendai at the upstream of the tidal reach. The model is given in the following form:

$$y_{SEN}(t) = a(t)y_{SEN}(t-1) + b(t)[y_{YUD}(t-T) - y_{YUD}(t-T-1)] + w(t) \quad (2)$$

where  $y_{SEN}(t)$  is water level at Sendai in meters,  $y_{YUD}(t-T)$  is water level at Yuda in meters,  $t$  is time in hour,  $T$  is time lag of flow between Yuda and Sendai in hours,  $a(t)$  and  $b(t)$  are ARMAX coefficients and  $w(t)$  is noise term as defined above. This is an autoregressive model of order 1, with the differenced upstream water level at Yuda as the exogeneous input. The time lag  $T$  is estimated from the water level data to be three hours, hence the differenced water level serves as a signal for the next three hours trend in the water level at Sendai. This ARMAX model is similar to the one reported by Ngan and Russell (1986).

The recursive application of the Kalman filter requires estimates of the system noise  $Q$  and observation noise  $R$ . Both the diagonal matrix  $Q$  and scalar  $R$  are assumed to be time-invariant, but the model coefficients are allowed to be time-variant, as modelled explicitly in the state equation. To allow time-variant model coefficients, the diagonal elements of  $Q$  are taken as the square of the elements of  $\hat{x}(0|0)$ , which are the least-square estimates of the model coefficients, for convenience sake. The scalar

observation noise  $R$  is assumed equal to the variance of the residuals which resulted from the least-square fit of the model to the given observed water levels.

## RESULTS AND DISCUSSION

The modelling and prediction techniques are applied to two sets of flood data on August 3-5, 1971 and September 10-13, 1976 at Kumisaki, Takaeomote and Sendai Gauging Stations. The forecasting performance of the Kalman filter is shown when the noise variances of the ARMAX and periodic coefficients are zero. For Kumisaki and Takaeomote, the misfits between predicted and observed water levels confirm that the mean  $M_y$  and amplitudes and phases of the 12-hour and 6-hour periodicities are not time-invariant. This assumption of time-variant system is also true during nontyphoon periods. Comparing the prediction results for Sendai between time-variant and time-invariant coefficients, it can be concluded that better forecasts can be obtained if the ARMAX coefficients are assumed time-variant. With time-variant coefficients for both models, the results of one-hour ahead predictions of water levels show that the prediction performance is satisfactory, with the predicted behaviour of the water levels following the observed one.

From the results of the prediction of the rate at which the water level rises at Sendai Gauging Station, it is found that there is a pronounced effect of the errors between predicted and observed water levels on the prediction of the rate of water-level rise. This means that if the ARMAX model overestimates or underestimates the water level, the rate will be overestimated or underestimated as well. It has been concluded that accurate prediction of the rate at which the water level rises is rather a difficult task using the present ARMAX model, and it has been recommended that confidence limits should be incorporated in the forecasted rate of water-level rise.

The identifications of  $M_y$ ,  $A_i$  and  $B_i$  show that the temporal variations in the model coefficients suggest that they are not constants but functions of time. The passage of flood in the lower end of the tidal reach is satisfactorily described by  $M_y$  and  $A_1$  and  $B_1$  of the semidiurnal tidal frequency component, with small contribution from  $A_2$  and  $B_2$  of the 6-hour oscillation. It is shown that the variation of the  $M_y$  is the most dominant, which is what one would expect during the passage of flood. Since  $A_2$  and  $B_2$  are relatively very small, it is possible to exclude the 6-hour frequency component.

The results of the identification of the coefficients of the ARMAX model  $a$  and  $b$  show that the variation of the autoregressive coefficient is bigger than that of the upstream input, which is what one would expect from the physical interpretation. At Yuda, the water level is largely influenced by the releases from the Tsuruta Dam so the upstream term is relatively stable, compared to the autoregressive term which has to account for the more variable tidal oscillations.

## CONCLUSIONS

It has been shown that a periodic function model with one or two frequency components can satisfactorily represent the behaviour of water level at locations where the influence of tidal motion is dominant, and an AR(1) model with exogeneous term can be applied at locations where the shape of flood hydrograph predominates. MEM spectral analysis has been used to decide which model is to be used at certain locations in the tidal reach. Moreover, this study has shown that mean  $M_y$  and periodic coefficients  $A_i$  and  $B_i$  ARMAX coefficients  $a$  and  $b$  should be allowed to vary with time by assuming proper noise variances to these parameters, when implementing Kalman Filter.

## REFERENCES

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