# I - 489 AN APPROXIMATE ANALYSIS OF IMPEDANCE MATRIX FOR RIGID FOUNDATIONS

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#### 1. INTRODUCTION

In the analysis of linear three-dimensional soil-structure interaction problem, boundary element methods (BEM) have been extensively used to calculate the impedance matrix, the dynamic force-displacement relationship associated with the degrees of freedom of the foundation. A modification to the solution of BEM is proposed here, by which the computational cost can be reduced due to a possible lessening of the number of sources. The procedure is based on an approximate solution of the equivalent rigid-body displacement.

#### 2. STATEMENT OF PROBLEM

A massless rigid three-dimensional foundation, occupying volume V', is perfectly bonded to the soil along the interface S. The soil, occupying volume V, is assumed to be a viscoelastic half-space (Refer to Fig.(1)). The response of the foundation is expressed by the generalized force  $F_0 = \{F_{0z}, F_{0y}, F_{0z}, M_{0z}, M_{0y}, M_{0z}\}^T$ , and the generalized displacement  $U_0 = \{U_{0x}, U_{0y}, F_{0z}, \Theta_{0x}, \Theta_{0y}, \Theta_{0z}\}^T$ , consisting of three translational and three rotational elements about a reference point  $(x_0, y_0, z_0)$ . The displacement and the traction field of the soil  $U(x) = \{U_x(x), U_y(x), U_z(x)\}^T$  and  $U(x) = \{U_x(x), U_y(x), U_z(x)\}^T$  must satisfy the Navier equations of motion, the conditions of vanishing traction on the surface of the half-space and the conditions:

$$U(x) = \alpha(x) U_0 \qquad F_0 = \int_S \alpha(x)^T T(x) dS(x) \qquad x \in S, \qquad U(x) \to 0 \qquad x \to \infty$$
 (1)

where  $\alpha(x)$  indicates the rigid-body influence matrix.

The solution of this mixed boundary-value radiation problem is expressed by the impedance matrix  $K_0$ , which relates  $F_0$  and  $U_0$  as

$$F_0 = K_0 U_0 \tag{2}$$

### 3. APPROXIMATE SOLUTION OF EQUIVALENT RIGID-BODY MOTION

When the source is applied within V', the formulation of BEM is given by

$$\int_{S}^{\nu} H(x,y)^{T} U(x) dS(x) = \int_{S} G(x,y)^{T} T(x) dS(x)$$
(3)

where G(x,y) and H(x,y) indicate displacement and traction Green functions respectively. Discretizing surface S, the displacement and traction field can be represented by a finite number of nodal values  $\{U\}$  and  $\{T\}$ , respectively. Then calculating the boundary integration numerically, equation 3 reduces to

$$[G]^T\{T\} = [H]^T\{U\} \tag{4}$$

Matrix [G] and [H] correspond to the value of the boundary integrals with respect to the Green functions and shape functions of the discretized displacement and traction field. From equation 4, the impedance matrix can be easily obtained. As BEM is based on representing the radiation displacement field as resulting from a set of sources, usually it is necessary to utilize a large number of sources.

Instead of claiming boundary conditions, we may decompose the resulting displacement U(x) into two parts,  $U_1(x)$  which satisfies the condition  $U_1(x) = \alpha(x) U_0$  at  $x \in S$ , and the remaining deformation  $U_2(x)$ . If the decomposition is made in such a way that the rigid-body displacement  $U_0$  corresponds to the resultant generalized force  $F_0$ , the relation between  $F_0$  and  $U_0$  will leads to the impedance matrix. In this case,  $U_0$  turns out to be the equivalent rigid-body displacement. An acceptable approximate solution of the equivalent rigid displacement may be obtained by the weighted average technique as

$$U_0' = \{ \int_{\mathcal{S}} \alpha(x)^T \alpha(x) dS(x) \}^{-1} \int_{\mathcal{S}} \alpha(x) U(x) dS(x)$$
 (5)

This solution has been used effectively for evaluating foundation input motion from the knowledge of the free-field motion and the solution of the impedance matrix. (1) (2)

## 4. RESULTS AND COMPARISON

To validate the proposed approach, the impedance matrix for cylindrical foundations embedded in a layered viscoelastic half-space has been calculated in the frequency domain. The geometry of the foundation and material property of the soil are presented in Fig.(2). Results are compared with those obtained by FEM and BEM. It can be seen from Fig.(3) that The solutions of BEM agree well when the source number is sufficiently large, but diverge at high frequencies when the source number is small; while solutions of the present method are quite reasonable with only 2 sources.

#### REFERENCE

- 1. Iguchi, M. An approximate analysis of input motions for rigid embedded foundations, Trans. Architectural Institute of Japan 1982, 57, 315, 61-75
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