I-306 STEADY-STATE ENERGY FLOW IN SDOF-TMD COMPOSITE SYSTEM

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INTRODUCTION : Using concepts of energy input, flow, storage, and output or dissipation, the mechanism of Tuned Mass Damper(TMD) as dynamic damper for a single-degree-of-freedom (SDOF) system is explained. Argand diagrams of dynamic forces are used to demonstrate graphically the different channels of energy flow.

ELEMENTS IN DYNAMIC MODEL : Figure 1 shows the 2-DOF composite system with mass, dashpot, and spring elements. The subscripts 1 and 2 refer to structure and TMD, respectively. X_1 and X_3 are absolute displacements. $X_2 = X_3 - X_1$ is TMD displacement relative to structure. The equations of motion are :

$$M_1\ddot{X}_1+C_1\dot{X}_1+K_1X_1-C_2\dot{X}_2-K_2X_2 = F(t) \qquad (1)$$

$$M_2\ddot{X}_2+C_2\dot{X}_2+K_2X_2 = -M_2\ddot{X}_1 \qquad (2)$$

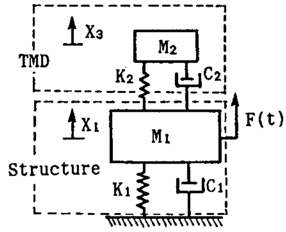
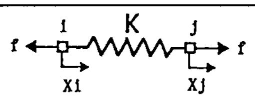
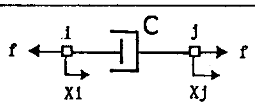
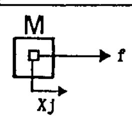
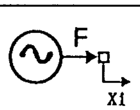


Figure 1. Dynamic model

The elements of dynamic model are described from energy view point in Table 1.

Table 1. Description of elements in dynamic model from energy point of view

ELEMENT	DIAGRAM & CONSTITUTIVE	ENERGY FUNCTION	ENERGY FORM
energy storage spring (K_1,K_2)	 $f=K(X_j-X_i)$ or $f=KX_r$ note $(X_r=X_j-X_i)$	$E=\int(-f)\dot{X}_i dt+\int(f)\dot{X}_j dt$ $E=-E_a+(E_a+E_b)$ where $E_a=\int KX_r\dot{X}_i dt$ $E_b=\frac{1}{2}KX_r^2$	Ea:energy flow through element(node j to i) Eb:energy storage in element
energy dissipator dashpot (C_1,C_2)	 $f=C\frac{d}{dt}(X_j-X_i)$ or $f=C\dot{X}_r$	$E=\int(-f)\dot{X}_i dt+\int(f)\dot{X}_j dt$ $E=-E_a+(E_a+E_b)$ where $E_a=\int C\dot{X}_r\dot{X}_i dt$ $E_b=\int C\dot{X}_r\dot{X}_r dt$	Ea:energy flow through element(node j to i) Eb:energy dissipated by element
energy storage mass (M_1,M_2)	 $f=M\ddot{X}_j$ or $f=M(\ddot{X}_i+\ddot{X}_r)$	$E=\int(f)\dot{X}_j dt=\frac{1}{2}M\dot{X}_j^2$ $E=E_a+E_b+E_c$ where $E_a=\frac{1}{2}M\dot{X}_i^2$, $E_b=\frac{1}{2}M\dot{X}_r^2$ $E_c=M\dot{X}_i\dot{X}_r$	E :total kinetic energy storage in element $\frac{d}{dt}(E_c)=M\ddot{X}_i\dot{X}_r+M\ddot{X}_r\dot{X}_i$
energy source harmonic exciter	 $F(t)=F_o\sin\omega t$	$E=\int F(t)\dot{X}_i dt=E_a+E_b$ $E_a=\omega F_oX_{io}\sin\theta\int\sin^2\omega t dt$ $E_b=\omega F_oX_{io}\cos\theta\int\sin\omega t\cos\omega t dt$ note $X_i(t)=X_{io}\sin(\omega t-\theta)$	Ea=energy feed through node i Eb=energy flow back and forth through i

ARGAND DIAGRAM OF FORCES (Fig.2) : The force vectors in diagrams are labeled by their respective amplitudes indicated by the subscript o. The dynamic equilibrium of forces at any instant is represented by equilibrium of real components. The force vectors, keeping the phase angles among them, are thought to rotate at an angular speed equal to the circular frequency of harmonic exciter F(t) (see also Ref. 1).

Only force components parallel to the velocity vector (shown in dotted arrow, for reference) do a net work over a full cycle. Force introduces energy when it is in-phase with $\dot{X}(t)$; it extracts energy when in-phase with $-\dot{X}(t)$. When 90^o phase different with $\dot{X}(t)$, the force merely shifts energy from one storage to another.

For the particular case shown in Fig.2, the $\dot{X}_1$ and $\dot{X}_2$ vectors have a phase difference of $\theta_1 = 90^o$, corresponding to the most effective condition of energy absorption (Ref.2). Assuming a mass ratio of $r = M_2/M_1 = 0.05$, and damping ratios of $\beta_1 = C_1/2\sqrt{K_1M_1} = 0.01$ and $\beta_2 = C_2/2\sqrt{K_1M_1} = 0.20$, this most effective condition is obtained when :

$$\delta = \omega/\sqrt{K_1/M_1} = 1-r/2 = 0.975 \qquad (3)$$

$$\Omega = (\sqrt{K_2/M_2})/(\sqrt{K_1/M_1}) = 1-r/2 = 0.975 \qquad (4)$$

ENERGY FLOW PATHS : For greater clarity in visualization of energy flow, two Argand diagrams are drawn for TMD, one with $\dot{X}_1$ as reference, the other with $\dot{X}_2$ (note that $\dot{X}_3=\dot{X}_1+\dot{X}_2$). Since $\dot{X}_1$ and $\dot{X}_2$ are 90^o phase different in this condition, the energy flow pattern itself is considerably simplified.

Two types of energy flow may be noted, one is energy flowing back and forth between energy storage elements. Examples are :

$$\frac{1}{2}K_1X_1^2 \longleftrightarrow \frac{1}{2}M_1\dot{X}_1^2 + \frac{1}{2}M_2\dot{X}_1^2$$

$$\frac{1}{2}K_2X_2^2 \longleftrightarrow \frac{1}{2}M_2\dot{X}_2^2$$

The other type has energy flowing in one direction. Energy is fed by the force F(t). Some of it is dissipated (output) to the environment through dashpot C_1; some flows through spring K_2 to be stored in $M_2\dot{X}_1\dot{X}_2$ (by the action of $M_2\ddot{X}_2$). At the same time some energy is channeled from $M_2\dot{X}_1\dot{X}_2$ (by the action of $M_2\ddot{X}_1$) and sent to dashpot C_2 to be eventually dissipated to the environment.

CONCLUDING REMARK : The above presentation clarifies the energy flow in element level of SDOF-TMD system as set in condition of most effective energy absorption. It reveals that the spring element (K_2) and dashpot element (C_2) in TMD not only do their respective basic functions, i.e., energy storage and dissipator, but also serve as energy channel.

REFERENCES : [1] Den Hartog, J.P.:MECHANICAL VIBRATIONS. 4 th ed., McGraw-Hill, NewYork, 1956

[2] Fujino, Y., Warnitchai, P. and Ito, M.:SUPPRESSION OF GALLOPING OF BRIDGE TOWER USING TUNED MASS DAMPER. J. of the Faculty of Eng., Univ. of Tokyo,B, Vol.XXXVIII, No.2 (1985).

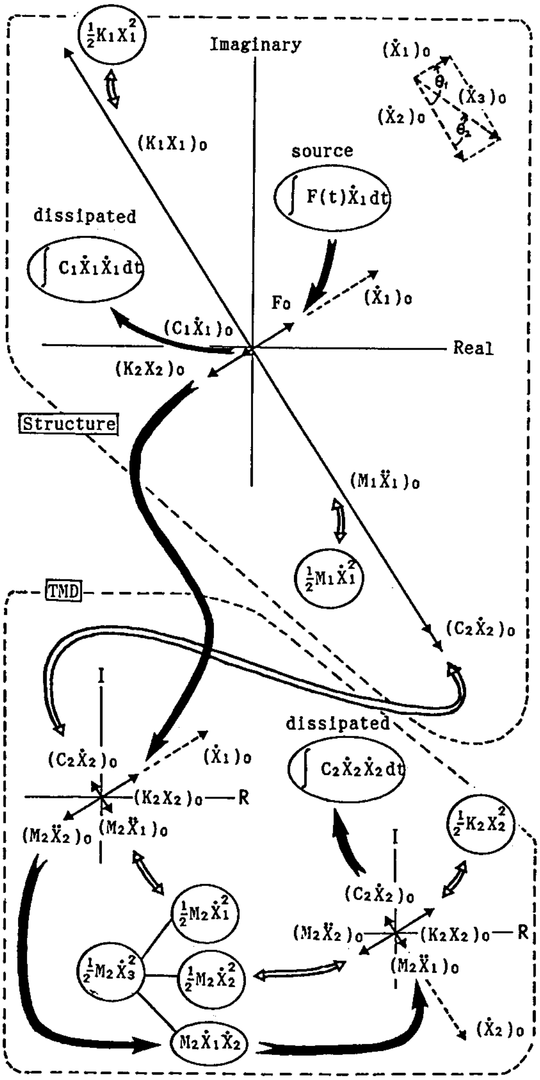


Figure 2. Argand diagrams and energy flow for SDOF+TMD system where r=0.05, Ω=0.975, δ=0.975, β_1=0.01, β_2=0.20