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SHAPE OPTIMUM DESIGN OF TRUSSES

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INTRODUCTION: The optimization of nodal coordinates in discrete structures is one of the most difficult tasks in structural optimization. The literature on shape optimization is rather sparse. Saka[1] presented a method which obtains the optimum joint locations while employing stiffness constraints for the structure as a whole. The stress constraints are complex and nonlinear functions of nodal coordinates and joint displacements.

This paper presents a contribution to the shape optimization of structures. The stiffness equations are written for each member independently, rather than the structure as a whole. This helps to reduce nonlinearity of constraints.

FORMULATION OF DESIGN PROBLEM: If stiffness equations are considered seperately for each individual member, rather than the structure as a whole, additional constraints in the form of nodal equilibrium equations need to be considered. Continuity at nodes is ensured by defining unique values of displacements, in the global coordinate axes, at each node. Further, the stress constraints are linear functions of member axial force and cross sectional area. The resulting nonlinear optimization problem is solved by the Lagrange multiplier method. The design criteria is that the stresses and/or displacements in a structure should not exceed certain permissible values.

Selecting an appropriate optimality condition, the design problem may be formulated as:

Minimize $W = W(A,X_c)$ subject to:

nodal equilibrium constraints: $[\sum_{i=1}^{n^j} (F_{ij})]_j - P_j = 0 \; ; j = 1, \ldots, N;$ ...(1)

member stiffness constraints: $F_{ij} - K_{ij}(A_{ij}, X_{ci}, X_{cj}, X_{di}, X_{dj}) = 0; ij = 1, \ldots, NM$ (2)

stress constraints: $|F_{ij}| - \sigma_{ip} A_{ij} < 0; \; i = 1, \ldots, NM$ ...(3)

joint displacement constraints: $-\Delta < \{X_{dj}\} < \Delta$ ...(4)

joint coordinate constraints: $X_l < \{X_{cj}\} < X_u$ ...(5)

non-negativity constraints: $A, \{X_{ci}\} > 0$ ...(6)

where A= vector of unknown areas, P_j= given external load vector at joint j, X_{dj} and X_{cj}= vector of joint displacements and coordinates respectively, at joint j; X_l and X_u= lower and upper bounds on X_c; σ_p and Δ= permissible values imposed on stresses and displacements, respectively; F_{ij}= force in the member lying between nodes i and j; N= total number of joints; NM= total number of members; and n^j= total number of members connected to joint j.

For determinate structures, only Eqns. (1), (3), (5), and (6) may be considered. However, such a consideration for an indeterminate structure gives an incompatible structure under the zero loading state; prestressing of members shall have to be employed in the construction phase. The designer may weigh the additional cost of prestressing against the saving of material and select the final design. If the objective function is taken as the weight of the structure,

$W = \sum_{i=1}^{NM} \rho_i A_i L_i$ where ρ_i, A_i = material density and cross-sectional area, respectively, of member i. The member length L_i is expressed in terms of joint coordinates.

An example stiffness constraint in Saka's[1] formulation for the two-bar truss shown in Fig. 1 is:

$$E\{[A_1L_1^{-3}x_1^2 + A_2L_2^{-3}(x_2-x_1)^2]x_{d1} + [A_1L_1^{-3}x_1y_1 - A_2L_2^{-3}(x_2-x_1)y_1]y_{d1}\} = H;$$

$L_1=(x_1^2+y_1^2)^{1/2}$; $L_2=((x_2-x_1)^2+y_1^2)^{1/2}$; and permissible stress constraint for member 1 is: $x_1L_1^2x_{d1} + y_1L_1^2y_{d1} < \sigma_{1p}/E$; compared to this, member stiffness constraint for member 1 in the proposed formulation is: $F_1 = EA_1L_1^{-2}(x_1x_{d1} + y_1y_{d1})$; and permissible stress constraint is: $F_1 - \sigma_{1p}A_1 < 0$; where x_1 and y_1 = the coordinates and x_{d1} and y_{d1} = the horizontal and vertical displacements, of node 1.

DESIGN EXAMPLE: The bridge shown in Fig.2 was considered as an example for design. The horizontal and vertical displacements of the joints were limited to 10 mm and 20 mm, respectively. Allowable stress in tension is $0.14kN/mm^2$, and in compression the lesser of $0.14kN/mm^2$ or $0.154\pi^2EA_i^{1.5}/L_i^2$ [1,2]. The modulus of elasticity is $210kN/mm^2$. Joints 5 ,and 6 are on the symmetry axis. Joints 1, 3, 7, and 9 are allowed to move only horizontally. Both the approaches mentioned earlier were applied for design. The optimal solution by both methods differ significantly in geometry and are listed in Table 1..

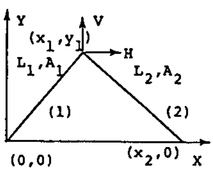

Fig.1 Two-bar truss

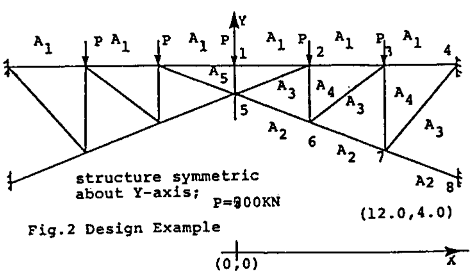

Fig.2 Design Example

Table 1. Optimal solution

Variable	Initial design	Design 'a'	Design 'b'
x_{c2}	4.000	6.000	6.000
x_{c3}	8.000	9.972	11.500
y_{c5}	8.500	9.000	9.000
x_{c6}	4.000	5.498	5.932
y_{c6}	7.000	8.133	7.830
x_{c7}	8.000	10.937	11.500
y_{c7}	5.500	5.244	4.487
A_1	3000.0	272.0	103.5
A_2	3000.0	4569.1	4976.9
A_3	3000.0	100.0	100.0
A_4	3000.0	1859.4	2054.8
A_5	3000.0	1428.6	1428.6
Volume	$29.02x10^4$	$15.84x10^4$	$17.08x10^4$

coordinates in meters; volume in mm^3; area of cross-section in mm^2; Design'a'- member stiffness equations not considered; Design'b'- member stiffness equations also considered;

CONCLUSIONS: The proposed method does not suffer from the weakness of repetitive analysis of the structure during the optimization process. Optimum shape can also be determined for multiple loading on the structure. The method offers the flexibility to cope with a variety of engineering and architectural requirements like a joint of a truss being restricted to move along a fixed line.

Table 2. Bounds on variables.

Variable	Lower bound	Upper bound
x_{c2}, x_{c6}	0.10	6.00
x_{c3}, x_{c7}	6.00	11.50
y_{c5}	0.00	9.00
y_{c6}, y_{c7}	0.00	9.00
A_1... A_5(mm^2)	100.0	none

coordinate values are in meters;

REFERENCES: 1.Saka, M.P., "Shape optimization of Trusses," Journal of Structural Division, ASCE, Vol. 106, No. ST5, Proc. Paper 15437, May, 1980.
2.Manual of Steel Construction, American Institute of Steel Construction, 1970.