I-278 NONLINEAR MIXED-DISCRETE STRUCTURAL OPTIMIZATION

Dept. of Agr. Eng., Kyoto Univ. ○H. M. Amir
Dept. of Agr. Eng., Kyoto Univ. T. Hasegawa

Introduction:- *Present report aims to solve Mixed-Discrete optimization problem with the solution code[1,2] originally proposed for all discrete variable problem with some modifications. Detail of the solution code can be available elsewhere only the modifications are added. Finally, effectiveness of the solution code is presents with example problems.*

MIXED DISCRETE PROGRAMMING PROBLEM

For Mixed-Discrete programming problem the optimization problem is defined as:

$$\min \quad F(X) \quad \text{------------------} \quad (1)$$

$$\text{subject to} \quad g_i(X) \geqq 0.0 \quad i \in M \quad \text{----} \quad (2)$$

$$X = x_1, x_2, \ldots\ldots, x_n = \begin{bmatrix} x^d \\ x^c \end{bmatrix} \quad \text{--------} \quad (3)$$

$x^d \in R^d$ = feasible subset of discrete variables

$x^c \in R^c$ = feasible subset of continuous variables

$i \in M$ = the set of constraint indices.

One of the sets R^c and R^d may be void. If R^c is void the problem is a complete discrete variable problem; if R^d is void, it is a complete all continuous variable problem. The problem is solved by converting Eq. 1 to 3 into a sequence of unconstrained problem by use of interior penalty function.

PROPOSED METHOD Present method is based on the idea to treat a continuous variable in the discrete sense with enough small values of resolutions. The small perturbations are taken in such a way that it mostly resembles the usual increments of the continuous variables. This idea in the analysis, therefore, resumes the properties of continuous variables and at the same time can be handled in the sense of discrete variables. The modifications performed in the respective part of the algorithm are as follows.

a) The IGD vector, GM(X), at a base point is developed as usual except the modification in the gradient approximation at X.

$$G(X) \equiv \begin{bmatrix} g_d(x) \\ g_c(x) \end{bmatrix} = \begin{bmatrix} \dfrac{f(x + (\Delta x_d)_i) - f(x)}{(\Delta x_d)_i} \\ \dfrac{f(x + (\Delta x_c)_i) - f(x)}{(\Delta x_c)_i} \end{bmatrix} \qquad (4)$$

where Δx_d and Δx_c are resolutions for discrete and continuous variables, respectively. $\Delta x_c = 0.01 x_c$, some other appropriate values may also be used.

b) The only modification to the Glankwhamdee's proposed SSI technique is the addition of Δx term to its equations, and are made as follows (Eq. 5a &5b). In a two dimensional search process, for mixed-discrete variable problem the SSI will turns to as of Fig. 1.

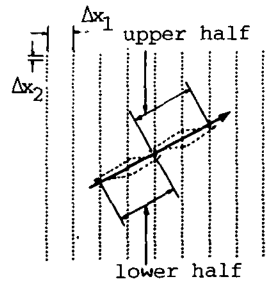

Fig.1 The SSI

$$X(k)_i = (X)_i - I\left\langle (K/2 - k + 1)\, dr_i/dx \right\rangle \Delta x_i$$

$$i = 1, 2, 3, \ldots, n; \quad k = 1, 2, 3, \ldots, K/2 \qquad (5a)$$

$$X(k)_i = (X)_i + I\left\langle (k - K/2)\, dr_i/dx \right\rangle \Delta x_i \qquad (5b)$$

$$i = 1, 2, 3, \ldots, n; \quad k = K/2+1,\ K/2+2, \ldots, K$$

c) Modified Rosenbrocks Orthogonalization Procedure, MROP, is further modified to suit to Mixed-Discrete optimization. The only modification is the addition of Δx term to the equation to find a new point along a direction of the orthogonal set.

$$(XT)^T = (X)^T + \lambda \cdot \Delta X\, S_i^{(j)} \quad \text{-------} \quad (6)$$

$$\Delta X = (\Delta x_d \quad \Delta x_c)$$

EXAMPLE PROBLEMS Two problems, **i) A Hatch cover problem** previously discussed elsewhere; and **ii) a Mill building structure problem** are considered for this purpose.

i) Hatch cover problem:-It is a two-variable problem as shown in Fig. 2. A hatch opening of l_0= 600.0 cm, is to be covered by 60.0 cm wide box of beam, of aluminum with cross section as shown in the figure. Free variables of the problem are, $x_1 = t_f$ = flange thickness; and $x_2 = h$ =the beam height. The objective function and constraints for this problem are given below.

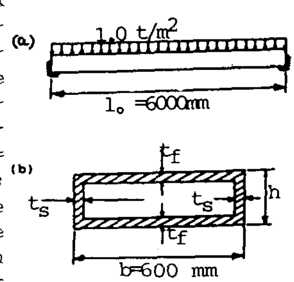

Fig.2 Hatch cover

$$F(X) = F(x_1, x_2) = x_1 + 120 x_2 \qquad (7)$$

$$\text{subject to} \quad g_1(X) = 1.0 - \frac{\sigma_b}{\sigma_{b,max}} \geqq 0.0 \qquad (8)$$

$$g_2(X) = 1.0 - \frac{\tau}{\tau_{max}} \geqq 0.0 \qquad (9)$$

$$g_3(X) = 1.0 - \frac{\delta}{\delta_{max}} \geqq 0.0 \qquad (10)$$

$$g_4(X) = 1.0 - \frac{\sigma_b}{\sigma_k} \geqq 0.0 \qquad (11)$$

$E = 700000\ kg/cm^2$; $\sigma_{b,max}=700\ kg/cm^2$

$\sigma_b = 4500\ kg/cm^2$; $\sigma_k = E\ t_f^2/1000\ kg/cm^2$

$\tau_{max} = 450\ kg/cm^2$; $\tau = 1800/x_1\ kg/cm^2$

$\delta_{max} = 1.5\ cm$; $\delta = 56.2\ 10^4/(E\ x_1\ x_2^2\ kg/cm^2$

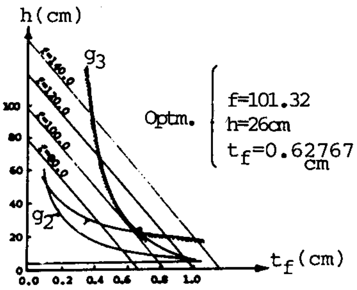

Fig.3 Constraints & Obj. func.

Results- Graphical solution for this problem is as of Fig. 3. In this research, variable x_1 is taken as discrete variable with the discretization pattern; x_1=1.0,2.0,····. Variable x_2 is taken as continuous variable with the resolution $(\Delta x_c)_2=0.01 \times x_2$. The optimum weight obtained is $F(X)^*$=101.843 with x_1^* = 27 cm and x_2 = 0.6237 cm. Profiles of PF(X,r) and F(X) function are shown in Fig. 4. From this results, it can claim that optimum result obtained is almost a global optimum; and the values taken by x_2 prove that handling a continuous variable in the discrete sense works well.

ii) Mill building structure problem:-

The mill building structure as shown in Fig. 5 with the dimension and loading condition is taken into account. The total weight of the structure is the objective to be minimized. The problem is designed under stress and displacement constraints.

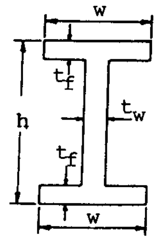

Fig.6 Cross section

Member 1 to 11 are restricted to take only discrete sections and 12 to 15 are assumed to be made of cross section of Fig 6. The problem is a symmetrical one, with member 1&11 form group 1, (G-1); similarly, G-2, G-3, G-4, G-5 and G-6 are consists of members 2, 6&9; 5&8; 3,5,7&10; 12&15; and 13&14, respectively. The modulus of elasticity, E; the specific weight, ρ and the yield point of stress, F_y are $3x10^4$ ksi, 0.2836 lb/in^3, and 26 ksi, respectively. Constraint equations are formulated according to the AISC specifications

TABLE 1. Results for Example problem 2

ITEMS	AD-sol.	MD-sol.	AC-sol.
$PF(X,r)^*$ lbs	10853.9	10827.3	10650.1
$F(X)^*$ lbs.	10849.5	10826.3	10649.7
Iteration	7	10	11
Func. evaluation	705	2163	4603
CPU time in sec.	11.55	35.44	77.09

Results:- Profiles of values of PF(X,r) and F(X) function for mixed discrete way of solution is given in Fig. 7. For comparison purpose, optimum results are given in Table 1. It is found that dealing a continuous variable in the discrete sense, works robustly. Proposed algorithm can conveniently be applied to All-discrete, All-continuous and Mixed-Discrete variable problems.

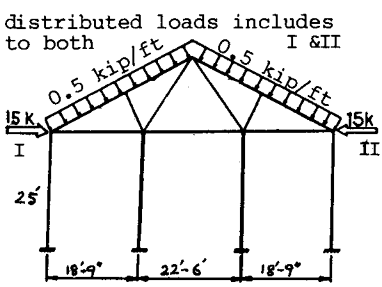

Fig.5 Main structure for Mill building (End elevation)

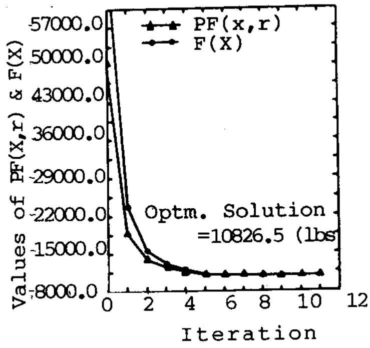

Fig.7 Profiles of PF(X,r) & F(X) function (MD-Sol)

References 1) Amir, H. M. and Hasegawa, T.:Nonlinear Discrete Structural Optimization; 42nd National Annual Conference of JSCE, Hokkaido, pp. 534-536, 1987. 2) Amir, H. M. and Hasegawa, T. :Nonlinear Discrete Structural Optimization, Proc. of JSCE, No. 392/I-9, 1988.