# I-10 ON THE WORST MODE OF IMPERFECTIONS FOR LOAD CARRYING CAPACITY

Univ. of Tokyo, Member , Fumio Nishino
Univ. of Tokyo, Student, Wibisono Hartono
Univ. of Tokyo, Member , Hideyuki Horii

INTRODUCTION : Imperfection sensitive structures such as thin-walled cylindrical shells under uniform axial compression have substantially lower carrying capacity compared with the bifurcation load of the perfect structures. This load carrying capacities vary widely because of small unintentional differences in the initial shapes of the structures. At present, the design of cylindrical shells is based on the theoretical critical load modified by empirical reduction (knock down) factors for each kind of loading [1].

This paper presents a numerical method to obtain the initial shape of a discretized structure which gives its lowest carrying capacity under a given norm of imperfections.

NUMERICAL PROCEDURE : Typical equilibrium paths of an imperfection sensitive and corresponding perfect structures are shown in Fig. 1 by the solid and dashed lines, respectively, where $f^c$ = loading intensity; $x$ = generalized position vector; and $x^0$ = generalized position vector at the initial state. Critical points of both systems are marked as solid and open dots, respectively. Superscript c indicates the values corresponding to the first critical point and additional bar sign indicates the value of the perfect system. The worst mode of imperfection can be obtained as a solution of the following optimization problem,

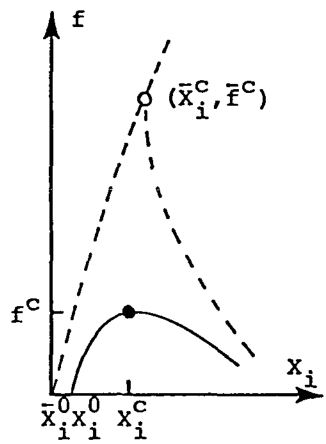



Fig. 1 Perfect and Imperfect Equilibrium Paths of Imperfection Sensitive Structure.

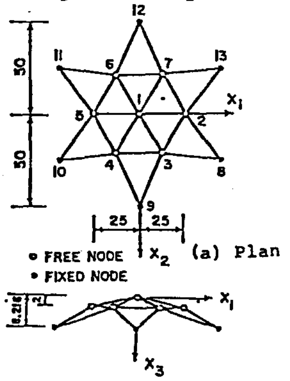
Minimize $f^c$

subject to

$$f^c f_i = K_i(x^c, x^0); \ \text{Det}\ K_{i,j} = 0; \ \|(x^0 - \bar{x}^0)\| = r \qquad (1)$$

where $f_i$ = loading pattern vector; and $K_{i,j}$ = tangential stiffness matrix. The first constraint is the equilibrium equation at the first critical point, the second one is the condition that must be satisfied at a critical point and the last constraint is the norm of initial imperfection where r is a given value. The unknowns of this optimization problem are $f^c$, $x^c$ and $x^0$.

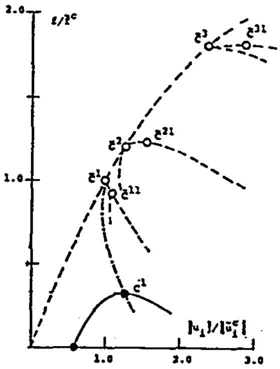
NUMERICAL EXAMPLE : A reticulated truss (Fig. 2) is selected to simulate the behavior of a spherical shell. The truss is subject to vertical loads at each node with the same intensity except at node 1 where only half intensity is applied. The equilibrium paths of the perfect truss are shown by dashed lines in Figs. 3a-e and the critical points are marked by open dots. The abscissa is the norm of displacements of all nodes and the ordinate is the loading intensity, both being non-dimensionalized by the corresponding value at the first critical point. The optimum solution corresponding to selected modes at the critical points which gives $f^c/\bar{f}^c$ equal to one third is marked as solid dots and the corresponding equilibrium paths are plotted by solid lines in the same figure. $\Delta$ indicates critical points which are not the solution and the chain lines show the position of the critical points for various value of r. It is noted that optimum solution can be either a stationary or a bifurcation point. The global minimum can be obtained as the lowest loading intensity of all solutions for the same value of r.

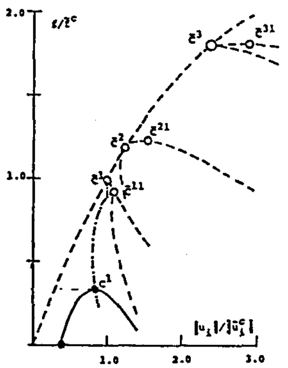


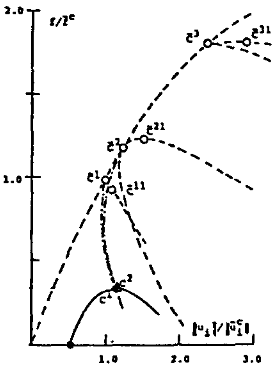
(a) Plan

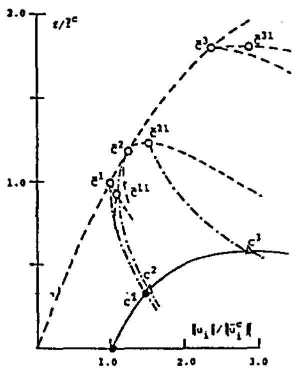
(b) Elevation

Fig. 2 Reticulated Truss


(a) $\bar{c}^1$ Mode


(b) $\bar{c}^{11}$ Mode


(c) $\bar{c}^2$ Mode


(d) $\bar{c}^2$ Mode

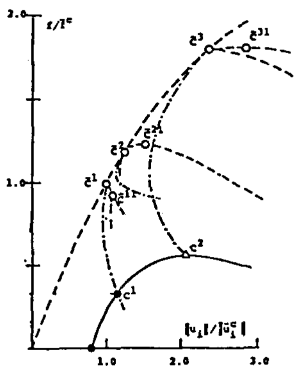

(e) $\bar{c}^3$ Mode

Fig. 3 Load Displacement Relation of Perfect and Optimum Imperfect Reticulated Truss.

<u>CONCLUSION</u> : A numerical procedure to obtain the lowest load carrying capacity of an imperfect sensitive structure is presented. For a given r, the solution consists of the location of the first critical point and the initial shape of imperfection. Although it is demonstrated using a truss example, this procedure can be applied to other imperfection sensitive structures.

<u>REFERENCES</u> :

1. Brush, D.O., and Almroth, B.O : Buckling of Bars, Plates and Shells, Mc. Graw Hill, Tokyo, 1975.