# V－260 MATHEMATICAL MODELING OF SEGGREGATION IN PIPE FLOW OF CONCRETE

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## INTRODUCTION

Seggregation is one of the major problems encountered in handling of fresh concrete. The level of seggregation may vary from seggregation of water to seggregation of aggregate in fresh concrete and it may depends on various factors. This paper describes an attempt which has been taken to simulate the seggregation of aggregate in concrete pumping by considering concrete as a two phase material.

## OUTLINE OF PIPE FLOW ANALYSIS

Since there is evidence that the concrete flows in a pipeline in the form of a plug separated from the pipe wall by a lubricating layer[1], one dimensional analysis is carried out. An elemental disk of diameter as same as the pipe diameter is considered in the analysis. This elemental disk contains both aggregate and paste phases which are idealized as two bodies moving on each other having the same thickness of the elemental disk. Assuming this idealization, cross sectional area of each phase has been calculated.

Kinematic flow equations for aggregate and paste phases are obtained (eq(1) and eq(2)) by applying Newtons law to two phases taking into account of all the forces acting on each phase as shown in Fig.1 .

for aggregate phase,

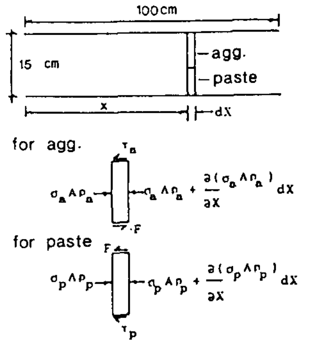
$$-\frac{\partial(\sigma_a A\rho_a)}{\partial X}dX - \tau_a + F = D_a A\rho_a\, dX\left\{\frac{\partial V_a}{\partial t} + V_a\frac{\partial V_a}{\partial X}\right\} \text{ —— (1)}$$

for paste phase,

$$-\frac{\partial(\sigma_p A\rho_p)}{\partial X}dX - \tau_p - F = D_p A\rho_p\, dX\left\{\frac{\partial V_p}{\partial t} + V_p\frac{\partial V_p}{\partial X}\right\} \text{ —— (2)}$$

where

$A$ – total cross sectional area of the pipe
$F$ – resisting force acting between aggregate and paste
$V_a, V_p$ – average velocity of aggregate phase and paste phase
$\sigma_a, \sigma_p$ – axial stress acting on aggregate and paste
$\tau_a, \tau_p$ – wall friction acting on aggregate and paste
$\rho_a, \rho_p$ – volume ratio of aggregate and paste
$D_a, D_p$ – specific gravity of aggregate and paste




Fig(1) Forces acting on agg. phase and paste phase

Continuity equations for both phases are derived and obtained as,

$$\text{for agg. : } \frac{\partial(AV_a\rho_a)}{\partial X} + A\frac{\partial\rho_a}{\partial t} = 0 \text{ —— (3)}$$

$$\text{for paste : } \frac{\partial(AV_p\rho_p)}{\partial X} + A\frac{\partial\rho_p}{\partial t} = 0 \text{ —— (4)}$$

And assuming incompressibility,

$$\rho_a + \rho_p = 1 \text{ —— (5)}$$

Aggregate stress ($\sigma_a$) may depend on the aggregate concentration and paste stress

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and it is assumed as,

$$\sigma_a = f(\rho_a, \sigma_p) \text{--------} (6)$$

The resistance between aggregate and paste (F) should be proportional to the volume ratio of aggregate, relative velocity of agg. with respect to paste and the viscosity (s) of paste. In this analysis, F is assumed as,

$$F = s\ g(\rho_a)h(V_p - V_a) \text{--------} (7)$$

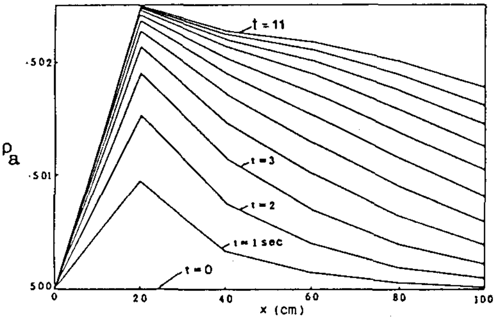
The total frictional resistance to the flow of concrete can be assumed to be the combination of friction between paste and wall and friction of aggregate in contact with the pipe wall. These forces may depend on the state of saturation of concrete[2]. Therefore two cases have been considered in the analysis, i.e. saturated state and unsaturated state. The frictional resistance between aggregate and pipe wall($\tau_a$) is considered as a function of $\rho_a$ and $\sigma_a$ in the unsaturated state and in the saturated state this frictional force is assumed as a function of $\rho_a$ only. Likewise in the case of paste phase ,wall friction($\tau_p$) is considered as a function of $\rho_p$ and $\sigma_p$ in unsaturated state while in saturated state it is assumed as a function of $\rho_p$ only.

## ANALYTICAL PROCEDURE

After writing the kinematic and continuity equations (Eq. 1,2,3 and 4 ) in the finite difference form, those equations are solved with the material models (i.e. F ,$\tau_a$ and $\tau_p$ ) by using Newtons iterative method and obtained values for six unknowns, i.e. $V_a$ ,$V_p$ ,$\sigma_a$,$\sigma_p$, $\rho_a$ , $\rho_p$, at each section.

## RESULTS AND DISCUSSION

From the analytical results (Fig.2), it can be seen that aggregate volume ratio is changing along the pipe line with time, indicating possible seggregation. The pressure variation along the pipe line is linear for saturated state while for unsaturated state it is an exponential curve (Fig.3). Analytical results indicate that the wall friction plays a major role not only in determining the required inlet pressure but also in material seggregation. And wall friction has the same degree of importance as the resistance between aggregate and paste (F). Extensive experimental studies are necessary to apply this theory to actual situations.




Fig(2) Volume ratio of agg. Vs distance

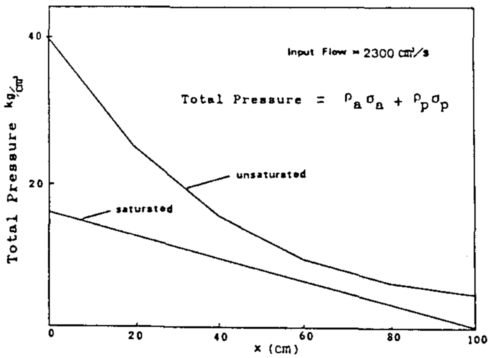



Fig (3) Total Pressure Vs Distance

## REFERENCES


[1] Browne, R.D., Bamforth, P.B., "Tests to Establish Concrete Pumpability," ACI JOURNAL, May 1977, pp. 193-203.

[2] Ede A.N., "The Resistance of Concrete Pumped Through Pipelines," Magazine of Concrete Research, Nov. 1957, pp. 129-140.