V-64 Fundamental Study on Constitutive Relations for a Cracked Reinforced Concrete Panel in the Mixed Mode of Displacements

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1.INTRODUCTION

The crack strain method in FEM analysis of a reinforced concrete structure is considered to be a very powerful means to incorporate material nonlinearities of various kinds in calculations. Although crack strain in a discontinuous solid is defined in various ways (1) (2) (3), crack strain has a natural relation to the damage tensor, which expresses the degree of damage of material from the intact condition. In this paper, the fourth rank damage tensor is defined in terms of crack strain and the reinforcement tensor is defined in terms of stress increment due to the reinforcement. The general constitutive equations for the composite material are developed for a two dimensional stress field using these tensors.

2.DEFINITION OF THE DAMAGE TENSOR FROM THE CRACK STRAINS

The damage tensor of the fourth rank may be defined in the following form to write stress reduction from the intact condition.

$$\Delta\sigma_{ij} = -\Omega^{D}_{ijpq} D_{pqmn} \varepsilon_{mn} \tag{2.1}$$

where $\Delta\sigma_{ij}$ is the reduction of the nominal stress due to damage in the solid from the intact condition. As $\Omega_{ijpq} = \Omega_{jipq} = \Omega_{jiqp} = \Omega_{ijqp}$, matrix expression for Eq.(2.1) is written as

$$\{\Delta\sigma\} = -[\Omega]_D [D]_c \{\varepsilon\} \tag{2.2}$$

Similarly, the reinforcement tensor may be written as

$$\{\Delta\sigma\} = [\Omega]_R [D]_c \{\varepsilon\} \tag{2.3}$$

Using Eq.(2.2) and Eq.(2.3), the general constitutive equation can be derived as

$$\{\sigma\} = [I - \Omega_{D1} - \Omega_{D2} - \cdots + \Omega_{R1} + \Omega_{R2} + \cdots][D]_c \{\varepsilon\} \tag{2.4}$$

On the other hand, stress reduction from the intact condition may be written as

$$\{\Delta\sigma\} = -[D]_c \{\varepsilon\}_{cr} \tag{2.5}$$

Substitution of Eq.(2.5) into Eq.(2.2), we get

$$\{\varepsilon\}_{cr} = [D]_c^{-1} [\Omega]_D [D]_c \{\varepsilon\} \tag{2.6}$$

Hence, in view of $\{\varepsilon\}_{cr} = [\Lambda]\{\varepsilon\}$, the damage tensor is obtained as

$$[\Omega]_D = [D]_c [\Lambda] [D]_c^{-1} \tag{2.7}$$

Similarly if $[\Omega]_R$ equals $= [\phi][D]_c^{-1}$,the reinforcement tensor is obtained as

$$[\Omega]_R = [\phi][D]_c^{-1} \tag{2.8}$$

3.CONSTITUTIVE EQUATIONS OF CRACKED RC PANELS

Constitutive equation of RC members for both separation mode and frictional mode have been discussed(3), that is $[\Lambda]$ are derived.Behavior of cracked RC panels can be treated as the coupling of the behaviors of frictional mode and separation mode (see Fig.1). For both modes, the tensors of damage and reinforcement can be derived as follows from the intact condition, referring to Eq.(2.5)(2.6)(2.7),and Eq(2.8).

$$[\Omega]_A = [\Omega]^S_{D1} + [\Omega]_{D2} - [\Omega]_R = [D]_c\left[[S]^{-1} + [D]_c^{-1}\right]^{-1}([D]_c + [D]_s)[D]_c^{-1} + [\Omega]_{D2} - [\phi][D]_c^{-1} \tag{3.1}$$

$$[\Omega]_B = [\Omega]^F_{D1} + [\Omega]_{D2} - [\Omega]_R = [D]_c\left([F] + [D]_c\right)^{-1}\left(I - [\Omega]_{D2}\right) + [\Omega]_{D2} - [\phi][D]_c^{-1} \tag{3.2}$$

Here, $[\Omega]_D$ and $[\Omega]_R$ are the damage tensor due to cracking and the reinforcement tensor.

$[\Omega]_{D2}$ is another damage tensor which is used to consider compressive deterioration

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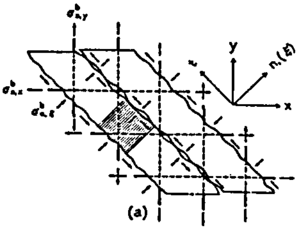
due to high intensity of compressive force.
Suffixes S, A and F,B denote separation mode and frictional mode respectively.
In stress conditions when the crack spacing is comparatively wide and frictional displacement at cracks occurs, the mixed mode of displacement takes place (see Fig.1). Basing on constitutive relations for the frictional mode and separation mode, we can develop the constitutive equations for the mixed mode. Obviously, at the boundary of two regions, the stress equilibrium should be satisfied. Also, the total elongation of the portion between two cracks is the sum of the elongation of each region of A and B, and the average strain of the total portion $\{\varepsilon\}_t$ is written as

$$\begin{Bmatrix}\varepsilon_n\\ \varepsilon_t\\ \gamma_{nt}\end{Bmatrix} = \begin{bmatrix}\eta&0&0\\0&1&0\\0&0&\eta\end{bmatrix}\begin{Bmatrix}\varepsilon_n\\ \varepsilon_t\\ \gamma_{nt}\end{Bmatrix}_A + \begin{bmatrix}1-\eta&0&0\\0&0&0\\0&0&1-\eta\end{bmatrix}\begin{Bmatrix}\varepsilon_n\\ \varepsilon_t\\ \gamma_{nt}\end{Bmatrix}_B = (\eta)(D)_c^{-1}(I-\Omega_A)^{-1}\{\sigma\}+(\zeta)(D)_c^{-1}(I-\Omega_B)^{-1}\{\sigma\} \qquad (3.3)$$

where, $\eta=\frac{\ell}{\ell_c}$

It should be noted that we can not have the frictional mode from the beginning since the crack initiation is always to the principal tensile direction and the first mode should be the frictionless mode. After a small crack width is formed, then the frictional mode or the mixed mode can exist.
Millard and Jonson (4) carried out an experiment corresponding to the mixed mode using the specimen shown in Fig.3(a),although this kind of experiments are very scarce. The experimental and calculated relations are compared in Fig.3(b),(c),(d),and(e).The calculated values give comparatively softer tendencies than the experimental ones. This result may be due to the assumption of η =0.5. The



(a)

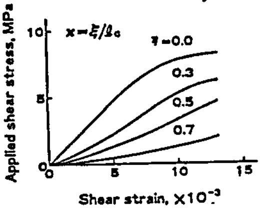
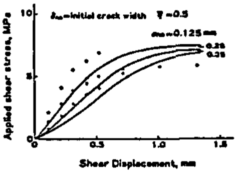


Fig. 2 The Variation of Shear Rigidity



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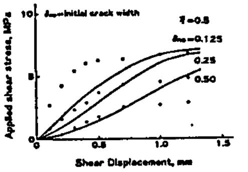

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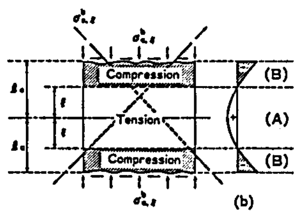

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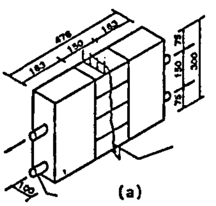
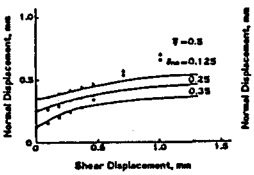
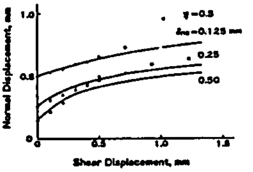


(a)



(c)



(e)

Fig.1 Stress Condition at a Crack in the Mixed Mode Displacement Condition

Fig.3 Shear Stress versus Shear Displacement and Normal Displacement Relations by Millard and Johnson (Ref.16)

differences in the shear rigidity due to the extent of the fraction of the region A of the total area are shown in Fig.3. The numerical calculations show that the greater the fraction of the region A, the softer is the shear rigidity.

4.CONCLUSION

The derived damage tensors make the nonlinear calculation much easier owing to the fact that Ω terms can be treated as initial strains or initial stresses to be accommodated in usual FEM programs, as we mentioned above, η values affects the behavior of the mixed mode and more detailed study may be necessary.

5. REFERENCES

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