III-56 Rapid Deformation of Granular Materials: Exact Binary Collision Model

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1. Introduction

The most careful kinetic theoretical approach following Jenkins and Richman[1] is employed in considering constitutive relations for two-dimensional flows of a dense collection of identical, rough, inelastic circular disks. A detailed account is given of the dynamics of binary collisions. The variation of the roughness coefficient with collision geometry is explored. It is shown how this geometrical dependence of binary collisions on the averaging scheme influences the accuracy of the constitutive relations and the findings there suggest development of a theory that can incorporate with a significant energy dissipation.

2. Exact Binary Collision Model

We are intersted in plane flows of a dense gas of identical, rough and inelastic circular disks. Dealing with a rapidly deoforming granular assembly it is recognized that collisional contribution to the constitutive quantities is dominant, and here the binary collision model is adopted in order to quantize such relations. It must be recognized that when two disks collide, there exist two modes of energy dissipation. The first is the energy dissipation associated with the normal component of the relative contcat velocity, and it is characterized by the coefficient of restitution. The second such dissipation mechanism is associated with the tangential component of the relative contact velocity. This is usually characterized by a parameter called the roughness coefficient (Jenkins & Richman[1]) and is usually treated as a constant.

Here we give a more detailed analysis on this most tractable model of collisional mechanism (that can successfully be used in the existing analytical frame) based on the observation made by Goldsmith[2]. He showed that the change in the tangential component of the relative velocity of the contact points depends on the geometry of the collision. In order to accomplish this we introduce a parameter called roughness coefficient, β , such that

$$\varepsilon_{\alpha\beta}\,\kappa_\alpha v'_\beta = -\,\beta\,\varepsilon_{\alpha\beta}\kappa_\alpha v_\beta$$

where $\varepsilon_{\alpha\beta}$ are the components of the alternating symbol; k_α are the components of the unit vector directed from the center of disk 1 to that of disk 2 at impact, and v_α and v'_α are the components of the relative contact velocities before and after the collision. (Fig.1)

The parameter β is assumed to vary between -1 and 1, and the range of interest here is between -1 and 0. Values of β there can be interpreted in terms of the friction between tangentially regid rough disks that slip during a collision. Coulom's law of friction is assumed to apply when disks

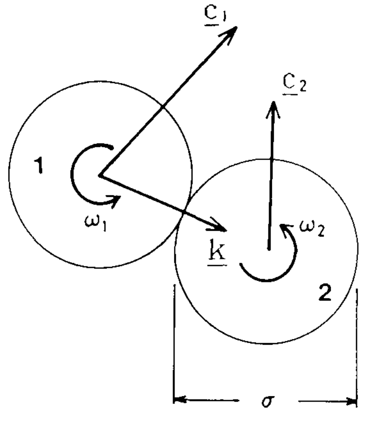

Fig.1 Two disks at impact

β can be related to the coefficient of friction by

$$\beta = -1 + \frac{\mu(1+e)(1+\kappa)\,|\cot\gamma|}{\kappa}$$

were γ is the collision angle between $\underline{v}$ and $\underline{k}$.

The statistical averaging scheme used in Jenkins and Richman[1] now is refined by casting the problem in terms of the above relation. Based on the variable roughness coefficient, β, the constitutive quantities are calculated in terms of the friction coefficient, μ, and coefficient of restitution, e, with the assumption that μ and 1-e are small. It is particularly interesting to note that the energy dissipation associated with the translational motion of the disks is far greater than what was expected to be. It is shown that when a collection of granular materials is rapidly deoformed, the majority of interactions between disks take place in such a way that the energy dissipation is most sever, and this may be attributed to the cause of the above finding. (Fig.2)

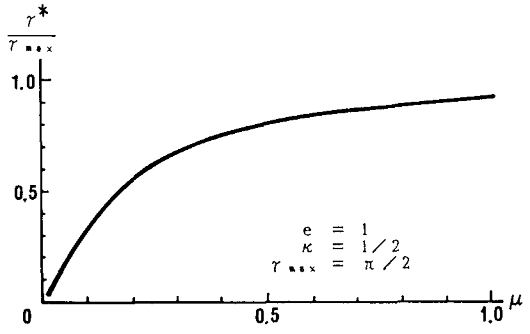

Fig.2 Probability of $\beta = 0$ collisions

3. Conclusion

Jenkins[3] remaked in his summary paper that when the average energy dissipation of a system is not small, the kinetic theory calculations [1),4),5),6)] all suggest the inadequancy of the isotropic Maxwellian. In particular, in their computer simulation Walton and Braun[7] show the distinctive anisotropic velocity distributions and conclude that the highly dissipative interactions among the disks are the cause of this anisotropy. It thus seems natural to value the findings here as an additional piece of information that might suggest a development of a theory with anisotropic Maxwellin velocity distribution function wich can successfully coope with dissipative systems.

4. References

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