I－323 MULTI-MODE GALLOPING OF BRIDGE TOWER WITH TWO CLOSELY-SPACED NATURAL FREQUENCIES

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INTRODUCTION

Certain bridge towers have in-plane vibration modes of closely-spaced natural frequencies. Galloping behaviour of such a structure has been studied by Fujino et al. [1], Phoonsak et al.[2]. They found that stable galloping of such a structure is motion either in lower frequency mode or in higher frequency mode depending upon the initial conditions. The multi-mode galloping is unstable. These results differs from the results obtained by Blevins et al.[3].

The present paper which is the continued work from Ref.1 and 2 attempts to analytically examine the conditions for stable multi-mode galloping and also to compare the results with observation from wind tunnel experiments.

EQUATIONS OF MOTION AND ASYMPTOTIC SOLUTION

Using the assumption of linear continuous structure and nonlinear quasi-steady wind force, the modal equations of motion for first mode, y_1 and second mode, y_2 can be expressed as

$$\ddot{y}_1+\omega_1^2 y_1 = \alpha_1\dot{y}_1+\alpha_2\dot{y}_1^2+\alpha_3\dot{y}_1\dot{y}_2+\alpha_4\dot{y}_2^2+\alpha_5\dot{y}_1^3+\alpha_6\dot{y}_1^2\dot{y}_2+\alpha_7\dot{y}_1\dot{y}_2^2+\alpha_8\dot{y}_2^3, \tag{1a}$$

$$\ddot{y}_2+\omega_2^2 y_2 = \beta_1\dot{y}_2+\beta_2\dot{y}_1^2+\beta_3\dot{y}_1\dot{y}_2+\beta_4\dot{y}_2^2+\beta_5\dot{y}_1^3+\beta_6\dot{y}_1^2\dot{y}_2+\beta_7\dot{y}_1\dot{y}_2^2+\beta_8\dot{y}_2^3, \tag{1b}$$

where $\alpha_1-\alpha_8$ and $\beta_1-\beta_8$ are function of mode shape, wind velocity and structural properties.

To solve Eqs. 1a and 1b, the nonlinear terms are assumed to be very small which can be characterized by a parameter ε. The solution is assumed as

$$y_1(t) = a_1(t)\cos\Omega_1+\varepsilon y_{11}(a_1,a_2,\Omega_1,\Omega_2),\quad y_2(t) = a_2(t)\cos\Omega_2+\varepsilon y_{22}(a_1,a_2,\Omega_1,\Omega_2), \tag{2a,2b}$$

where $\Omega_1 = \omega_1 t-\delta_1(t)$, $\Omega_2 = \omega_2 t-\delta_1(t)-\delta_2(t)$.

The variables a_1, a_2, Ω_1 and Ω_2 are assumed to be slowly varying functions of time t.

The case $\omega_1 \doteq \omega_2$ is considered. To express the closeness between these two natural frequencies, the detuning parameter σ is introduced as $\omega_2=\omega_1+\varepsilon\sigma$. Performing certain algebraic manipulations leads to the equations for steady-state response as

$$\dot{a}_1 = 0.5\,a_1\alpha_1+0.375\,\omega_1^2a_1^3\alpha_5+0.25\,\omega_2^2a_1a_2^2\alpha_7[1+0.5\cos 2\lambda] +0.375\,[\omega_1\omega_2a_1^2a_2\alpha_6+\omega_2^3a_2^3\alpha_8/\omega_1]\cos\lambda, \tag{3}$$

$$\dot{a}_2 = 0.5\,a_2\beta_1+0.375\,\omega_2^2a_2^3\beta_8+0.25\,\omega_1^2a_1^2a_2\beta_6[1+0.5\cos 2\lambda] +0.375\,[\omega_1\omega_2a_1a_2^2\beta_7+\omega_1^3a_1^3\beta_5/\omega_2]\cos\lambda, \tag{4}$$

$$\dot{\delta}_2 = \{-0.125\,[\omega_1^2a_1^3a_2\beta_6+\omega_2^2a_1a_2^3\alpha_7]\sin 2\lambda-0.325\,[\omega_1^3a_1^4\beta_5/\omega_2+\omega_2^3a_2^4\alpha_8/\omega_1]\sin\lambda -0.125\,\omega_1\omega_2\,a_1^2a_2^2\,[\alpha_6+\beta_7]\sin\lambda\}/(a_1a_2), \tag{5}$$

where $\lambda=\varepsilon\sigma t-\delta_2$, $\dot{\lambda}=\varepsilon\sigma-\dot{\delta}_2$.

Steady-state amplitude a_1 and a_2 and phase lag δ_2 can be obtained by applying the conditions that $\dot{a}_1=\dot{a}_2=\dot{\delta}_2=0$ in Eqs. 3, 4 and 5. Stability of the steady-state solution must be examined by considering the small perturbations at the solution point.

ANALYTICAL EXAMPLE AND EXPERIMENTAL COMPARISON

In Eq. 5, if both β_5 and α_8 are non-zero, both a_1 and a_2 must be non-zero in order that $\dot{\delta}_2=0$, i.e. only steady-state multi-mode galloping exist. These two parameters, β_5 and α_8, are non-zero for certain bridge towers, for example tower with unsymmetrically distributed mass or tower with irregular cross-section. Then, the bridge tower in Ref.1 which is modified by adding a small mass at one leg of the tower as shown in Fig.1 is employed as the case study.

Steady-state solutions and their corresponding stability of this tower are examined at various wind velocities. It was found that the solution of $a_1=0$ and $a_2=0$ is stable, i.e. no galloping, when the wind velocity is less than the onset wind velocities ($U<Ucr_1, Ucr_2$). It is very interesting to note that when $U>Ucr_2$ but $U<Ucr_1$,

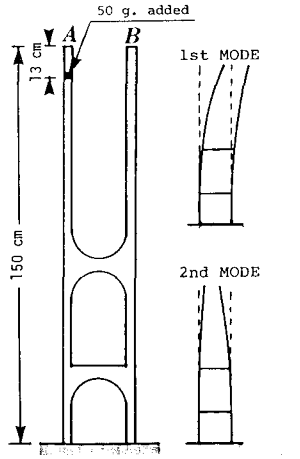

Fig.1 Bridge tower employed in case study.

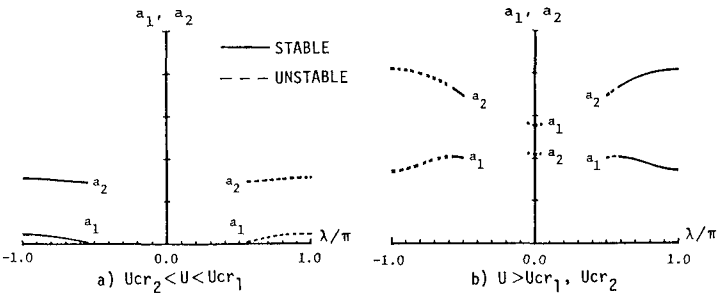

Fig.2 Steady-state multi-mode solution.

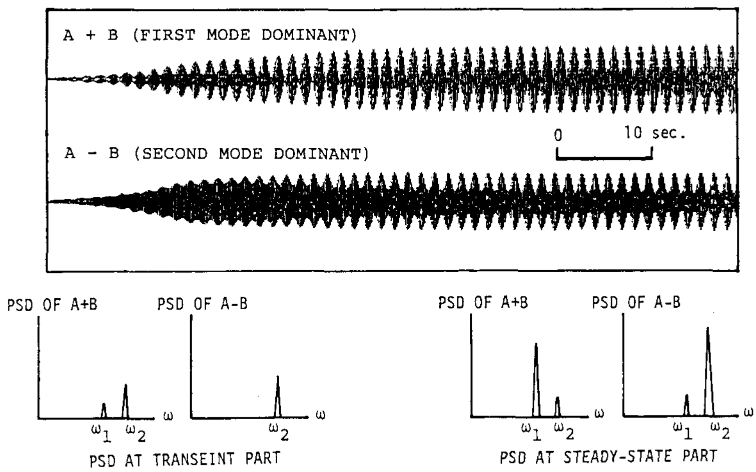

Fig.3 Time history response and power spectrum density (PSD) obtained from wind tunnel experiment.

only the multi-mode galloping is stable as shown in Fig. 2a. At higher wind velocities ($U>Ucr_1, Ucr_2$), the multi-mode galloping is stable as shown in Fig. 2b.

Fig. 2 indicates that in the multi-mode galloping the stable steady-state amplitude of the first mode is less than that of the second mode. This is because the structural damping of first mode is greater than that of second mode. It is also found that each modal amplitude slightly changes according to the value of λ. The stable solution depends only on the initial phase lag.

Galloping behaviour of this tower model was also experimentally studied in the wind tunnel. It was observed that only the multi-mode galloping was stable. Time history responses from the rest position and their power spectrum density at $U>Ucr_1$ and $U>Ucr_2$ are presented in Fig. 3. Initially, the tower oscillated mainly in the second mode motion. As time passed the first mode motion gradually increased along with the second mode until both reached the stable multi-mode motion. The response power spectrum density further verifies this phenomenon. With respect to the coexistence of two modes in galloping, the experimental results agree with the analytical results.

CONCLUSION REMARK

For tower of certain properties, such as unsymmetrically distributed mass or irregular cross-section, the analysis indicates that multi-mode galloping is stable. Modal steady-state amplitude slightly depends upon the initial phase lag between the two modes. Analytical results were verified by the wind tunnel experiments.

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