I-250 NONLINEAR DISCRETE STRUCTURAL OPTIMIZATION

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1. INTRODUCTION: In this report three different techniques namely, i) The integer gradient direction, IGD, ii) The subsequential search interval,SSI and iii) modified Rosenbrocks orthogonalization procedure, ROP, have hybridized to solve nonlinear discrete structural optimization problem. The necessary modifications of the techniques and their combination made it possible to overcome most of the practical difficulties usually encountered in a optimization problem.

2. DESCRIPTION OF THE TECHNIQUES

2.1 THE INTEGER GRADIENT DIRECTION, IGD.

The IGD first introduced by Glankwhamdee et al(1), and considered reasonably efficient and convenient, since it utilizes concepts of general gradient formation for continuous variables and search only over a set of discrete points. The IGD in a base point (X) can be calculated as follows.

a) Calculate G(X), the approximated gradient at (X):

$$G(X) = \frac{F(x+\Delta x)-F(x)}{\Delta x} \quad (1)$$

b) Calculate S(X), the normalized gradient direction at (X), $S(X)=G(X)/\|G(X)\|$

c) Calculate DR(X), the relative gradient direction at (X); $DR(X)=S(X)/|s|$; where 's' is is the smallest (absolute) elements of S(X).

d) Calculate GM(X), the integer gradient direction, IGD, at (X) by changing values in DR(X) to the nearest integer value. Now, any discrete point (XT) along GM(X) can be generated from the equation

$$(XT) = (X) + \lambda\, \Delta x\, GM(X) \quad (2)$$

in which λ represents the optimal step length along GM(X) to obtain (XT) from (X). Δx is a diogonal matrix of resolutions in which Δx_i is the resolution of x_i design variable.

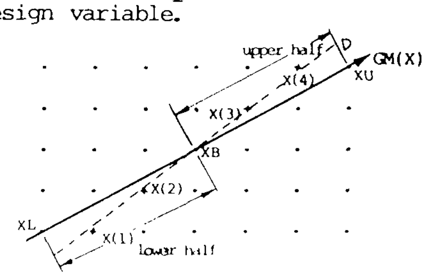

Fig.1 The subsequential search interval

Resolution of x_i, Δx_i, is the shortest distance between two discrete points on a line parallel to x_i axis.

2.2 THE SUBSEQUENTIAL SEARCH INTERVAL,SSI.

Purpose of the SSI is to search points in the vicinity of the base point and IGD that do not fall precisely on the line of search. The SSI in a two dimensional search space is illustrated in Fig. 1.

2.3 MODIFIED ROSENBROCKS METHOD, ROP

Rosenbrocks method with Gram-Schmidt orthogonalization procedure is a well known search method and it has the ability to identify and follow ridges. Detail of the method can be available some where else only the modifications are described here.

Each elements of the unit direction of ROP are transformed to the integer valued as of section 2.1. The respective step lengths λ_1, λ_2,...., λ_n associated with the orthonormal search directions $S_1^{(j)}$, $S_2^{(j)}$,..., $S_n^{(j)}$ in the jth stage must be a integer. The search begins by making a perturbation of $\lambda_i^{(j)} \cdot S_i^{(j)}$, (i=1 to start with), in the $S_i^{(j)}$ direction. If the search is a success the λ_i value must also multiply by a integer α value to keep the discrete points in its discrete nature. On the other hand, if the search deemed a failure, the λ_i value is multiplied by a integer factor, β, which is selected in in such a way that up to a certain distance along $S_i^{(j)}$ all points in the backward and forward side can be tested.

Again, if any of the mutual orthogonal directions fails to provide any improvement in a certain stage of ROP, then Gram-schmidt procedure fails to provide orthonormal direction. To avoid such difficulties, order of the unit directions are rearranged in such a way that the direction which shows no improvement are at the last of the order and the orthogonalization are carried out as usual. After completion, order of the unit directions are rearrange back to the original order.

3. OPTIMIZATION PROBLEM

Generalized optimization problem are of the form

$$\text{Min } F(X)$$
$$\text{subject to} \quad g_i(X) \geq 0;\ i=1,2,...n \quad (4)$$
$$X \geq 0$$

This constrained problem can be solved by converting them into a sequence of unconstrained problem by use of interior penastrained problem by use of interior penalty function and the problem formulation becomes

$$\min PF(X,r) = F(X) + r.G(X) \quad (5)$$

Where G(X) is the some function of $g_i(X)$; r = penalty function parameter. Usually

$$G(X) = 1/\sum_{i=1}^{m} g_i(X) \quad (6)$$

4. *EXAMPLE PROBLEM*

a) 4-bay, 1-storey plane frame:-

The problem is to minimize the weight of the frame shown in Fig.1. There are three loading conditions, 1) Vertical distributed load as shown in Fig. 1; 2) Vertical distributed loads as shown and a wind load of 10K acts from left to right at joint 2; and 3) vertical distributed loads as shown and a wind load of 10K acts from right to left at joint 9. The solution space is given in Table 1 and is limited to 14W standard sections. Only stress constraints are considered.

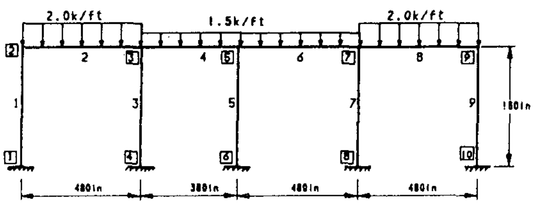

Fig. 2 Four bay, 1-storey plane frame

Table 1. Design space

X (1)	Designation (2)	A, in inch^2 (3)	I, in inch^4 (4)	S, in inch^3 (5)
1	W14x22	6.49	198	28.9
2	W14x26	7.67	244	35.1
3	W14x30	8.83	290	41.9
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
46	W14x665	196.00	12500	1150.0
47	W14x730	215.00	14400	1280.0

A: cross-sectional area; I: moment of inertia; S: section modulus

Note: 1 in=25.4mm; 1 in^2=645mm^2; 1 in^3=16400mm^3; 1 in^4=416200mm^4;

Each of the three loading conditions are treated as the independent alternate loads acting on the frame, therefore, there are 27 stress constraints. The effective length factor, K_f, for each of the columns are assumed to be 2.0.

4.1 RESULTS AND DISCUSSION:- The solution has started from the initial base point: (X)=[35 35 35 35 35 35 35 35 35]$_T$

The objective function value at the initial design is 64626.75 lbs., with penalty function parameter r_1 =4550.0. The PF. function value at the initial base point is 113427.3 lbs. The optimal PF. function value of this constrained problem obtained, PF(X,r)* =12928.4 lbs. and is reached when r_{11}= 0.004. Total calculation time, CPU=12.94 sec. Detail of the results are given in Table-2 in a summarized fashion and Fig. 3 shows the profiles of PF(X,R) and F(X) with the iteration number.

Table 2. Results for Example problem

Iteration (k)	Value of r_k	Starting $F(X,r)_k$ in lbs.	Optimum $F(X,r)^*_k$ in lbs.
1	4550.0	113427.3	91920.4
2	1137.0	45916.0	41347.6
3	284.4	26283.0	23889.6
4	71.1	18550.6	17317.9
5	17,7	15113.2	14456.7
6	4.44	13623.4	13457.7
7	1.11	13129.6	13050.6
8	0.28	12958.9	12958.9
9	0.07	12936.0	12936.0
10	0.017	12930.3	12930.0
11	0.004	12928.8	12928.8

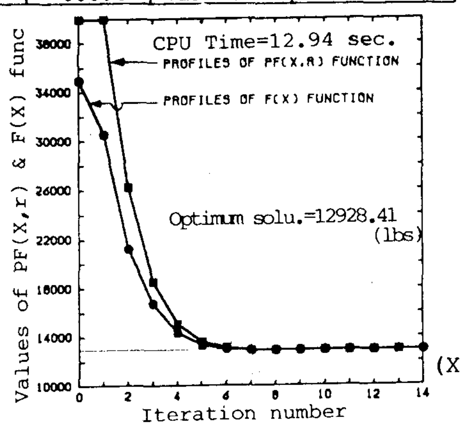

Fig.3 Profiles of PF(X,r) & F(X) function

Discrete way of structural optimization described here is robustly worthy. Results of the test problem reveals that very good optimum point has obtained. Effects of IGD from the true steepest descent direction mostly recovered by the SSI, resolution valley difficulties also over come by the modified Rosenbrocks method.

5. *REFERENCES:-*

1) Glankwhamdee, A., Liebman, J.S. and Hogg, G.L.: Unconstrained discrete nonlinear programming, Engineering optimization, Vol.4, pp.95-107, 1979.

2) Rao, S. S.:Optimization; Theory and applications second editions, Wiley eastern ltd. New Delhi, India, 1984