I-248 AN OPTIMIZATION PROCEDURE FOR PRELIMINARY STRUCTURAL DESIGN

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1. INTRODUCTION: A large number of works have been reported for optimum design of elastic structures. Most of the past works use all of the governing equations for the whole structure as the constraints for optimization. This makes the optimization problem highly nonlinear with respect to the variables for the objective function, thus making numerical analysis difficult. There are few works which treat the parameters to change shape of structures as variables for optimization.

In this paper, a method is presented in which only equilibrium equations and continuity conditions at the nodes of the discretized structure, and member stiffness equations are used as constraints for the optimization problem. This is the most important contribution of this study which reduces nonlinearity of constraints and makes shape optimization easier.

2. PROBLEM FORMULATION: For this optimization problem, the weight of the structure is selected as representative criterion of the objective function. Both the lengths and the areas of cross section of arbitrary selected elements, on which the weight depends, are taken as variables. The elements between the nodes are treated to be prismatic and each cross section is described by a single design variable.

To formulate the constraints, equilibrium equations and continuity conditions are written at each node of the discretized structure as well as the equilibrium equations for each element. The equilibrium equations are formulated considering the structure in the displaced configuration.

The proposed method is intended to take into account multiple loading conditions simultaneously. Because of this it is necessary to keep the stiffness of each individual element of the structure constant through the optimization process. For this purpose the member stiffness, $E(I/L)_{ij}$ or $E(A/L)_{ij}$, is expressed in terms of member end forces and displacements by utilizing the member stiffness equations and then kept constant for the different loading conditions on the same element ij. If the structure has to be optimized only for a single loading the optimization process becomes even simpler as the constraints to keep stiffness of each of the elements individually constant for successive loading cases is no longer required.

3. NUMERICAL EXAMPLE: The developed theory is demonstrated through an example on the preliminary design of a three span continuous beam, as shown in Fig. 1, where the span lengths and the cross sections of the members are optimized for minimum weight under given total length and two loading cases. The loading consists of concentrated loads, as given in Table 1. and shown in Fig. 1.

The structure is discretized into elements 1-2, 2-3, and 3-4.

The constraint equations are:

a. Equilibrium equations at the nodes 1,2,3, and 4 of the structure of Fig. 1.

b. Three individual equations for each of the elements 1-2, 2-3, and 3-4.

c. Consistency among the end forces and end displacements for each element.

d. Maximum flexural stress to be less than or equal to the allowable/or limit stress in bending at each of the nodes and at points of application of concentrated loads.

The objective function is:

$$\text{Minimize } \sum_{ij} \gamma A_{ij} L_{ij} ,$$

$ij=1-s1,\ s1-2, \ldots, s3-4.$

where A_{ij} and L_{ij} are the area of cross section and length of member ij respectively and γ is the density of the material. The ratio of the area of cross section to section modulus is assumed to be constant.

The results of optimization are summarised in Table 2.

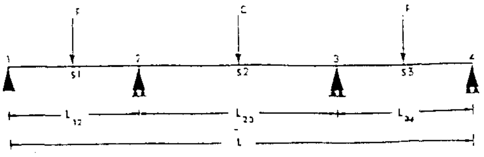

Fig. 1 Three span continuous girder optimized for minimum weight, under concentrated loads P, Q, and R acting at midspan.

Table 1. Loading on the beam of fig. 1

Load (in tons)	Loading Case 1	Loading Case 2
P	100.0	300.0
Q	200.0	150.0
R	300.0	100.0

Table 2. Results of Optimization

Parameter	Initial Solution	Optimal Solution
L (m)	17.50	17.50
L_{12} (m)	5.00	3.62
L_{23} (m)	7.50	7.79
L_{34} (m)	5.00	6.09
A_1 (cm^2)	430.00	360.99
A_{s1} (cm^2)	270.00	262.66
A_2 (cm^2)	500.00	373.80
A_{s2} (cm^2)	400.00	286.46
A_3 (cm^2)	500.00	410.30
A_{s3} (cm^2)	550.00	460.32
W (kg)	6083.75	4985.54

4. CONCLUSIONS: This study shows that the optimization problem for preliminary structural design can be solved by using simple constraints based on the equilibrium concept rather than governing equations for the whole structure.