I-108 INTERACTION BEWTEEN A MAIN CRACK AND MICROCRACKS

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1. INTRODUCTION: The presence of process zones consisting of large numbers of microcracks has been reported in brittle materials such as concrete, rocks and ceramics. The microcracking constitutes an important mechanism which affects the growth of the main crack. The interaction between the main crack and microcracks has already been studied theoretically in a number of investigations(2,3,4). They are based, however, on some unexamined assumptions or approximations. In the present study, a general formulation is presented and the accuracy of different approximate approaches is discussed.

2. FORMULATION: Consider a two-dimensional linear elastic solid containing a semi-infinite crack and an arbitrarily oriented microcrack, as shown in Fig.1. The exact solution is not available in general. Hence the method of pseudo-traction (1) is employed. The original problem is decomposed into three subproblems, each of which contains one single crack; see Fig.1.

The initial stresses $\sigma_0(x)$, $\tau_0(x)$ along the position of the microcrack are obtained from the near-tip stress field in subproblem 1. To find the stresses $\sigma(x_0)$, $\tau(x_0)$ along the position of the main crack in subproblem 2, the initial stresses $\sigma_0(x), \tau_0(x)$ and the pseudo-tractions $\sigma_p(x), \tau_p(x)$ are expanded into Taylor series as

$$\sigma_0(x)-i\tau_0(x)=\sum_{n=0}^{\infty}(\sigma_{on}-\tau_{on}i)(x/c)^n,\quad \sigma_p(x)-i\tau_p(x)=\sum_{n=0}^{\infty}(\sigma_{pn}-\tau_{pn}i)(x/c)^n. \quad (1)$$

By using the method of Muskhelishvili, $\sigma(x_0)$, $\tau(x_0)$ can be obtained explicitly in terms of unknown coefficients σ_{pn}, τ_{pn}. Then the solution to subproblem 3 is obtained with σ_{pn}, τ_{pn}. The pseudo-tractions, i.e. σ_{pn}, τ_{pn} are determined so that the boundary conditions of the original problem are satisfied. The stress intensity factors for both the main and the microcrack are then obtained according to the superposition principle. It is shown that σ_{pn}, τ_{pn} are the order of $(c/d)^{n+2}$, while σ_{on}, τ_{on} are that of $(c/d)^n$.

3. DISCUSSION: Firstly, a paticular case of collinear microcrack is considered. By using the present formulation, the stress intensity factors for the main crack and the microcrack are given by

$$K_I^{MA}=K_I^o\left(1+\frac{1}{4}\left(\frac{c}{d}\right)^2+\frac{23}{128}\left(\frac{c}{d}\right)^4+\ldots\right),\quad K_I^{MI}\Big|_{x=\pm c}=K_I^o\sqrt{c/2d}\left(1\mp\frac{1}{4}\left(\frac{c}{d}\right)+\frac{1}{4}\left(\frac{c}{d}\right)^2+\ldots\right) \quad (2)$$

which, as expected, coincide with Taylor expansion of exact solutions in terms of elliptical integrals.

For an arbitrarily oriented microcrack, the expressions become complicated. The first-order solution is obtained by retaining σ_{00}, τ_{00} and neglecting the pseudo-traction, which gives the first two terms of the stress intensity factor for the main crack and one term for the microcrack. The second-order solution is obtained by retaining $\sigma_{00}, \tau_{00}, \sigma_{01}, \tau_{01}, \sigma_{02}, \tau_{02}, \sigma_{po}, \tau_{po}$, which gives the first three terms for both the main crack and the microcrack. Some numerical results are shown in Figs.2 and 3 for the general cases.

Finally, approximate approaches appeared in the literature are discussed for the collinear microcrack.

(A) The method by Chudnovsky and Kachanov(2) corresponds to use $\sigma_{00}, \tau_{00}, \sigma_{po}, \tau_{po}$ and to use an additional approximation in subproblem 3 resulting in

$$K_I^{MA} = {}_I^0\left(1+\frac{1}{4}\left(\frac{c}{d}\right)^2+\frac{23}{128}\left(\frac{c}{d}\right)^4+\ldots\right),\quad K_I^{MI}\Big|_{x_1=\pm c}=K_I^0\sqrt{c/2d}\left(1+\frac{1}{4}\left(\frac{c}{d}\right)^2+\ldots\right) \quad (3)$$

(B) The method by Rose(3) corresponds to use σ_{00}, τ_{00} and σ_{po}, τ_{po} and to make an approximation in subproblem 2 leading to

$$K_I^{MA}=K_I^0\left(1+\frac{1}{4}\left(\frac{c}{d}\right)^2+\frac{2}{128}\left(\frac{c}{d}\right)^4+\ldots\right),\quad K_I^{MI}\Big|_{x_1=\pm c}=K_I^0\sqrt{c/2d}\left(1+\frac{1}{16}\left(\frac{c}{d}\right)^2+\ldots\right) \quad (4)$$

(C) The method by Hoagland and Embury(4) corresponds to use σ_{00}, τ_{00} and σ_{po}, τ_{po} without further approximations arriving at

$$K_I^{MA}=K_I^0\left(1+\frac{1}{4}\left(\frac{c}{d}\right)^2+\frac{17}{128}\left(\frac{c}{d}\right)^4+\ldots\right),\quad K_I^{MI}\Big|_{x_1=\pm c}=K_I^0\sqrt{c/2d}\left(1+\frac{1}{16}\left(\frac{c}{d}\right)^2+\ldots\right) \quad (5)$$

For the approaches (A)-(C), only the first two terms for the main crack and one term for the microcrack coincide with the exact solution. The approach (A), however, happens to give, for the collinear case, the third correct term for the main crack and the second correct average term for the microcrck. For an arbitrarily oriented microcrack, all the previous approximate methods fail to provide correct higher order terms.

4. CONCLUSIONS: In the present study, a general solution to the interaction problem of a main crack and a microcrack is obtained, and the inconsistency in some reported solutions is shown. The present method can be extended to include mutiple microcracks.

5. REFERENCES:

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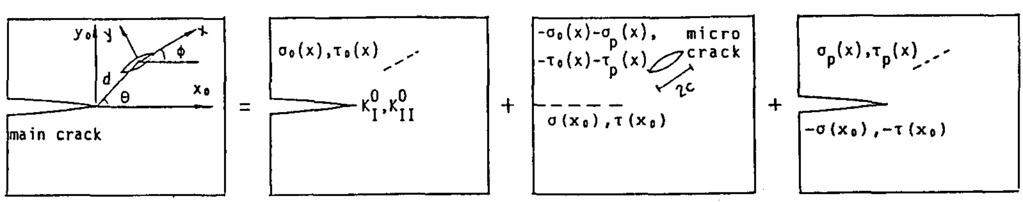

Original problem　　Subproblem 1　　Subproblem 2　　Subproblem 3

Fig.1 Decomposition of the original problem into three subproblems

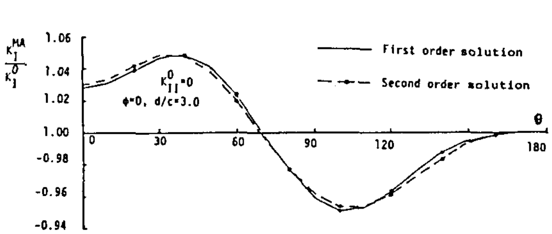

Fig.2 Variation of the stress intensity factors for the main crack

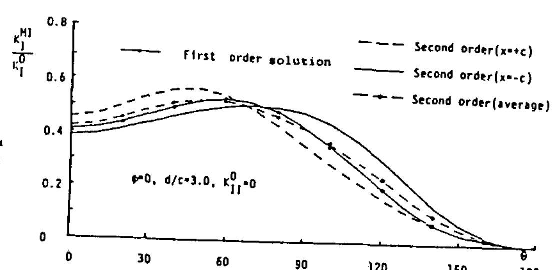

Fig.3 Variation of the stress intensity factors for the microcrack