I-107 **MECHANICAL PROPERTIES OF CRACKED SOLIDS : DOUBLY PERIODIC CRACKS**

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INTRODUCTION

The behavior of solids containing multiple cracks has been the subject of interest of numerous researches, especially those in the field of geomechanics. The response of such solids depends not only on the properties of the matrix surrounding the cracks, but also on the size, shape, orientation, and distribution of the cracks. As a simplified model for mathematical investigation, the problem of a solid having a doubly periodic rectangular array of cracks is considered, with the intention to clarify the inconsistency between the results of the previous works [1] and [2].

ANALYSIS

An infinite elastic plane containing a doubly periodic rectangular array of cracks is shown in Fig. 1. The solution for this solid subjected to the far-field stresses is given by the superposition of the solutions of a homogeneous problem and a subsidiary problem, as shown in Fig. 2. The homogeneous problem contains no cracks and is subjected to uniform stresses at infinity. In the subsidiary problem, the solid contains a doubly periodic set of cracks with no stresses at infinity but having the stresses prescribed on all the crack surfaces such that the boundary conditions along any crack k are given by

$$\sigma_y^k = -\sigma_y^\infty \ , \ \tau_{xy}^k = -\tau_{xy}^\infty \ , \ -c \leq x^k \leq c \ , \ y^k = 0 \ . \tag{1}$$

The subsidiary problem is solved by employing the method of pseudo-tractions proposed by [3], which makes use of the superposition principle to decompose the subsidiary problem into infinite number of sub-problems of a single crack (see [4]). The first-order approximate but explicit solution is obtained by assuming that the pseudo-tractions are constant along the crack surfaces. The stress intensity factors for mode I and II are given (see [4]) as

$$\frac{K_I}{\sigma_y^\infty\sqrt{\pi c}} = 1 + \frac{P_0}{\sigma_y^\infty} , \quad \frac{K_{II}}{\tau_{xy}^\infty\sqrt{\pi c}} = 1 + \frac{Q_0}{\tau_{xy}^\infty} , \quad \begin{Bmatrix} P_0 \\ Q_0 \end{Bmatrix} = \lim_{\substack{R\to\infty \\ C\to\infty}} \sum_{i=-R}^{R} \sum_{j=-C}^{C} \begin{Bmatrix} F(i,j) \\ G(i,j) \end{Bmatrix} . \tag{2}$$

The terms P_0 and Q_0 are expressed in terms of doubly infinite series of explicit functions F(i,j) and G(i,j), representing the interaction effects from all other cracks on any crack of interest. These summations are carried out by first considering an array of cracks of R rows by C columns and then increasing C and R with constant C/R. It is seen that for different values of C/R the series tend to different limits, which means that the doubly infinite series of Eqn.(2) are not convergent (see [4]). This kind of divergent series arises when the superposition principle is employed to account for the interactions in the doubly periodic structures.

However, it can be shown (see [4]) that, being expressed in terms of the average stresses $\hat{\sigma}_y$, $\hat{\tau}_{xy}$ evaluated along the middle portion of the crack array, the stress intensity factors given by

$$\frac{K_I}{\hat{\sigma}_y\sqrt{\pi c}} = 1 + \frac{c^2}{HW} H_I\left(\frac{H}{W}\right) , \quad \frac{K_{II}}{\hat{\tau}_{xy}\sqrt{\pi c}} = 1 + \frac{c^2}{HW} H_{II}\left(\frac{H}{W}\right) , \tag{3}$$

are unique.

For mode I, the results given by Eqn.(3) for small values of c/W are found to be close to those of [1] but different from those of [2]. In fact the analysis made by [1] does not involve the doubly infinite series since it deals with a rectangular plate containing a single crack, which is equivalent to the

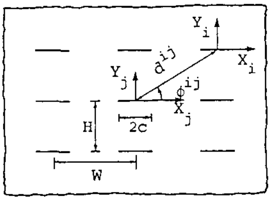

Fig. 1 A doubly periodic rectangular array of cracks

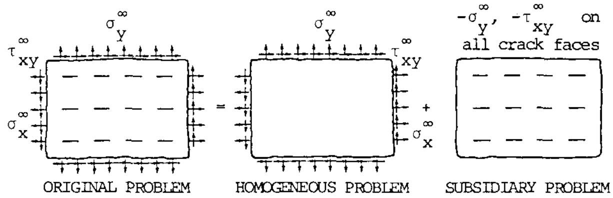

Fig. 2 Decomposition of an original problem into a homogeneous problem and a subsidiary problem

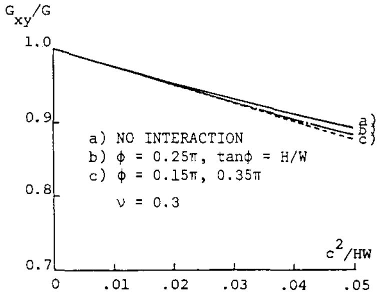

Fig. 3 G_{xy}/G as a function of the crack density.

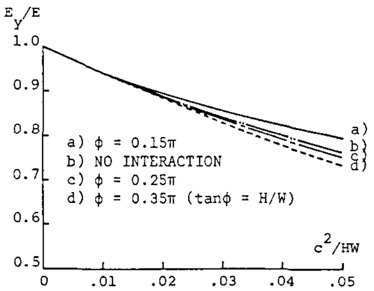

Fig. 4 E_y/E as a function of the crack density.

present problem by symmetry. On the other hand, the method used by [2] has its basis on the superposition, thus leads to the terms with doubly infinite summation in the formulation. The summations are evaluated in the vertical direction first, and the resulting integral equations are then solved numerically. Letting C/R be very small, which is equivalent to summing the series in the vertical direction first, Eqn.(2) gives the results close to those given by [2]. Therefore it seems that the solution obtained by [2] corresponds to Eqn.(2), which is not convergent.

Once the stress and the strain fields of the solid are known, the overall compliance of the solid can then be derived as a function of c^2/HW and H/W (see [4]). The effective moduli of the solid for the case of plane stress are shown in Figs.3 and 4.

CONCLUSIONS

The present study reveals that the difficulty in solving problems of doubly periodic cracks arises from the superposition principle. An approximate but explicit solution is derived. The stress intensity factors and the overall compliance are explicitly obtained as functions of the crack density and a parameter describing the geometry of the crack array.

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