I-45 ELASTO-PLASTIC FINITE DISPLACEMENT ANALYSIS OF THIN-WALLED OPEN CROSS SECTION MEMBERS AND FRAMES

Tohoku Univ. ○Khaled MAALLA
Tohoku Univ. Shigeru KURANISHI
Tohoku Univ. Tetsuo IWAKUMA

1. INTRODUCTION

Recently, a considerable amount of theoretical studies has been carried out on the post-buckling elastic behaviour and the spatial instability of thin-walled open cross section members. These studies are found to be excellent in predicting the elastic buckling load and the post-buckling behaviour as well. In the present study, an attempt to extend the solution to include the inelastic range of the material, is presented.

In contrast to Ref.4 in which yielding is assumed to be a function of the normal and shear stresses and Von-Mises criterion is employed, this method is based upon the non-consideration of the shear stress in the yield condition and this will render the solution much easier than the ones proposed by Ref.3 and 4.

In the following, the outlines of this method are given and a variety of simple problems are carried out to justify its applicability.

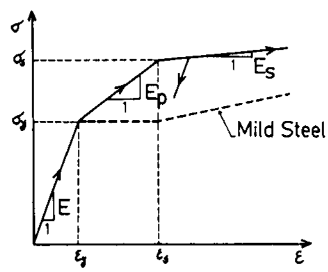

FIG.1 Stress-Strain Diagram

2. BASIC EQUATIONS

1)Stress-Strain Relationship

$$\dot{\sigma} = E_t(\varepsilon, \dot{\varepsilon})\,\dot{\varepsilon} \qquad (1)$$

where, Fig.1

$$E_t = \begin{cases} E & \text{if } \varepsilon \le \varepsilon_y \text{ or } \varepsilon > \varepsilon_y \text{ and } \varepsilon\dot{\varepsilon} < 0 \\ E_p & \text{if } \varepsilon_y < \varepsilon \le \varepsilon_s \text{ and } \varepsilon\dot{\varepsilon} > 0 \\ E_s & \text{if } \varepsilon > \varepsilon_s \text{ and } \varepsilon\dot{\varepsilon} > 0 \end{cases} \qquad (2)$$

2)Incremental Stiffness Equation

The incremental stiff. equation for the whole structure under consideration obtained after introducing the selected displacement functions into the expression of the virtual work and assembling, may be written for the(i+1)th step as,

$$[K_T(E_t, P_i(x))]\{\Delta r\} = \{\Delta R_i\} \qquad (3)$$

$$\{R_i\} - \{F_i\} = 0 \qquad (4)$$

where $[K_T]$ is the tangent stiff. matrix after i steps which is function of E_t (Tangent Modulus) and $P_i(x)$ (Stress Resultants which vary linearly through the element). $\{R_i\}$ and $\{F_i\}$ are, respectively, the ext. and int. force vectors existing in the equilibrated reference state. $\{\Delta r\}$ and $\{\Delta R\}$ are the incremental disp. and load vectors, respectively.

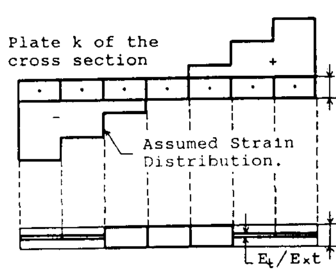

Fig.2 Transformed Area Concept

3. SOLUTION PROCEDURE

Since numerical integrals are necessary to evaluate the stress resultants and the cross sectional properties which both depend on the loading history, the cross section is divided into a convenient number of small segments parallel to its profil line. The strains are determined in the center of each segment and added to the previous ones. Then the tangent modulus is judged as explained in Eq.2. Therefore the thickness of the segment is modified by the modular ratio and the necessary properties of the partially yielded cross section which are assumed to vary linearly through the element, are evaluated in the extremities of each element. The tangent stiff. matrix can be formed and Eq.3 is solved. Then the stress resultants and the coordinate system are updated to solve for the next step. It must be noted that, when dealing with elastic materials, Eq.4 need not to be checked (provided that small increments are used) but whenever inelastic material is concerned, this equation must be checked in every step otherwise $[K_T]$ will deviate from its correct path and then there will be possibility of error.

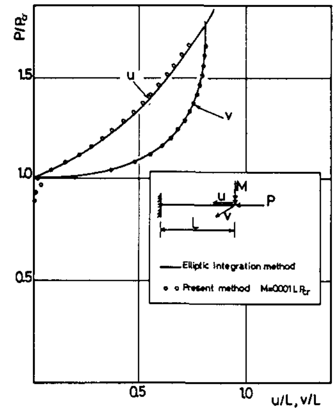

Fig.3: Large deflection of a buckled I section cantilever

3. RESULTS AND DISCUSSIONS

Three simple examples were examined numerically and compared to the available theoretical results in order to show the efficiency of this method. Fig.3 shows the solution by this method of the elastica problem. This solution is compared with the analytical one obtained by the elliptic integration. A very good agreement can be observed eventhough no iterations were performed. As a second example, an inelastic bending analysis was carried out mainly to show the effect of the strain hardening. The highest number of iterations was three and the results, as shown in Fig.4, seem to be very good. In the last example, a plastic buckling analysis is carried out. As shown in Fig.5, the max. load obtained from the present method is found to be 5.3% smaller than the one given by Perry Robertson formula.

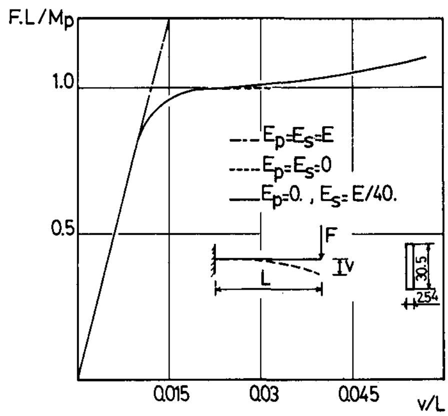

Fig.4: Inelastic bending of a cantilever

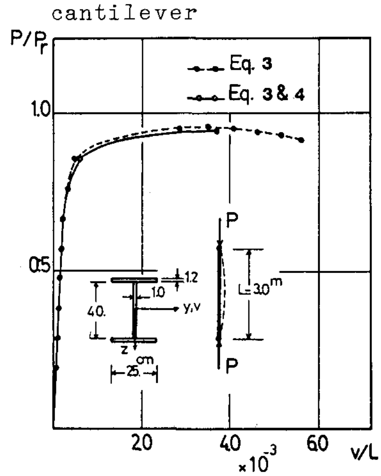

Fig.5: Inelastic Euler buckling of an I section column

4. CONCLUSION

This simple and easy method which can take into consideration the effect of strain reversal and residual stresses, seems to give satisfactory results whenever the ultimate and/or the elastic critical load of thin-walled open cross section structures(which are fundamental for safe and economical design), is required.

REFERENCES

1)Bazant,Z.P. and Nimeiri,M.E : Proc.ASCE, Vol.99, EM6, Dec.1973.
2)Liyanage,K. : Doctoral Dissertation, Univ. of Tokyo, 1985.
3)Epstein,M., Nixon, and Murray : Proc.ASCE, Vol.104, ST5, May 1978.
4)Sakimoto,T., Yamao,T., Kikuchi,R. and Sakata,T. : Proc.JSCE, No.362/I-4, Oct.1985.