I-12 ON THE WORST MODE OF IMPERFECTION OF GEOMETRICALLY NONLINEAR TRUSS

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1. INTRODUCTION : The stability a real structure can be considerably influenced by the existence of imperfections. The actual imperfections are difficult to evaluate, hence it is more important to know the worst mode of them in reducing the maximum carrying capacity to be taken into account in design purposes. This study presents an optimization procedure to obtain the worst mode of the imperfections.

2. OPTIMIZATION PROCEDURE : Considering systems with proportional loading [1] for simplicity, the optimization problem then can be expressed as

Minimize f

subjected to

$$ff_i = K_i(x_i,\xi_j) \quad \text{and} \quad \xi_j\xi_j = r^2 \quad (i = 1 \text{ to } n,\ j = 1 \text{ to } m) \tag{1a,b}$$

where K_i = i-th component of internal force vector; f, f_i = loading intensity and loading pattern vector, respectively; x_i= position vector; ξ_j= initial imperfection mode vector; r = norm of imperfections; and n,m = number of degrees of freedom and components of imperfection of the system, respectively. Equation 1a is the governing equilibrium equations of the imperfect system. Noting that the constraints are nonlinear equations with equality signs, the above optimization problem can be transformed by the Lagrange multiplier method into solving a system of nonlinear equations, consisting of Eqs. 1 and

$$\lambda_i\frac{\partial K_i}{\partial x_j} = 0 \ ,\ \lambda_i\frac{\partial K_i}{\partial \xi_j} - 2\lambda_{n+1}\xi_j = 0 \ ,\ \lambda_i f_i = 1 \tag{2a-c}$$

where λ_i (i = 1 to n+1) = the Lagrange multipliers. Equations 1 and 2 are (2n+m+2) nonlinear equations in terms of (2n+m+2) unknowns, consisting of f, x_i, ξ_i and λ_i. To reduce the number of equations, λ_i are eliminated by taking advantage that Eqs. 2 are all linear in term of λ_i. Selection of initial values and scaling play important role in solving this nonlinear equations. By giving a small value of r, the corresponding values of the perfect system can be used as the initial values. For a relatively larger values of r, those values then can be obtained by interpolation.

3. NUMERICAL EXAMPLES : The above optimization procedure is demonstrated in numerical examples of truss structures of which imperfections are given in the form of initial length imperfections and expressed as

$$L^{ip} = (1 - \xi_i)L^p \tag{3}$$

where L^{ip}, L^p = initial length of imperfect and perfect bar, respectively.

1) Consider the plane two bar truss as shown in Fig. 1. This truss is subjected to a vertical load at node 1. For $\gamma = 2.5$, the loading intensity and position of node 1 at the first bifurcation point of the perfect system are given in Table 1. Selecting $r^2 = 0.0008$, the value of the worst imperfections mode, loading and position of node 1 at the limit point which is the first critical point are also given in Table 1.

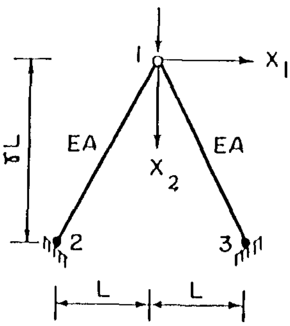

Fig. 1 Two Bar Plane Truss.

Table 1. First Critical Point of Perfect and Imperfect Two Bar Truss.

	Position of Node 1		Loading	Imperfection Mode r^2 = 0.0008	
	x_1/L	x_2/L	f/EA	$\xi_1(x10^{-2})$	$\xi_2(x10^{-2})$
Imperfect System	0.752	0.546	0.292	-2.463	1.390
	-0.752	0.546	0.292	1.390	-2.463
Perfect System	0.000	0.705	0.414		

2) A reticulated space truss as shown in Fig. 2 is analyzed as a more complex example. The truss is subjected to a vertical load at each node with the same intensity, except at node 1 where intensity of one half is applied [1]. After eliminating λ_i, the number of unknowns are reduced to 46. Fig. 2 shows the loading intensity vs. vertical position of node 1 for perfect (dashed line) and imperfect system (solid line) corresponding to the worst mode for $r^2 = 1.1478x10^{-5}$. Also shown in Fig. 2 by a dotted line is the lowest limit load vs r^2 relationship. At the state where the lowest loading intensity is one third of the corresponding value at the first critical point of the perfect system, the maximum of absolute values of ξ_i of all members is less than 0.13%.

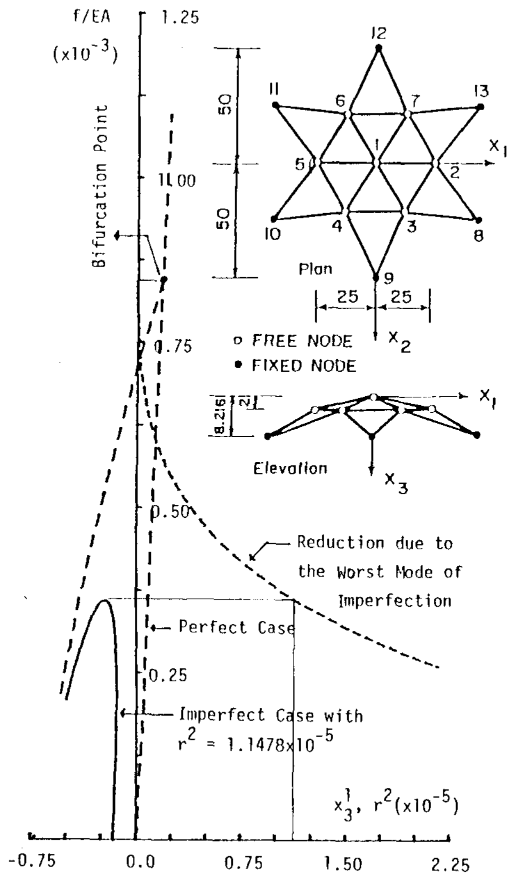

Fig. 2 Loading Intensity vs Vertical Position of Node 1 of Perfect and Imperfect System and the Lowest Limit Point vs r^2 of Reticulated Space Truss.

4. CONCLUSIONS

By employing Lagrange multiplier method, the optimization problem to obtain the worst mode of imperfection is transformed into solving a system of nonlinear equations which is easier to solve using the available software packages. Due to its simplicity, this procedure could be incorporated in practical design analysis.

5. REFERENCE

1. Hartono W., Nishino F., Fujiwara O. and Karasudhi P.: On Tracing Bifurcation Equilibrium Paths of Geometrically Nonlinear Structures , Proc. of JSCE, Struct. Eng./Earthquake Eng., Vol. 4, No 1, April 1987.