II-252

NUMERICAL SIMULATION OF A SHEAR DRIVEN FLOW IN A TEST RESERVOIR BY SGS MODELLING

Marko IVETIC, University of Belgrade
Yoshiaki IWASA, Kyoto University
Kazuya INOUE, Kyoto University

INTRODUCTION

Turbulent flow in a test reservoir caused by wind shear stress has been simulated numerically using space averaged time-dependent Navier-Stokes equations. For this type of flow, like for the most of the geophysical flows, experiments are difficult to conduct, and field measurements are very expensive (and often incomplete), so appropriate mathematical modelling has indispensible role in lake and reservoir management and water quality control. However, no matter how true mathematical modelling could be, without field measurements model cannot be tuned or confirmed.

The Navier-Stokes equations are filtered (or space averaged) instead of time averaging, as it is usual for obtaining the Reynolds' equations. The objective of this method is to remove small eddies from the flow field [Deardorf,1970], and to model them (Sub Grid Scale modelling). Derived equations are for large eddies , so the other name for the method is Large Eddy Simulation. It is observed that only large scale eddies differ significantly between flows, while small scales are nearly isotropic. Also flow properties are dominated by the influence of large, energy containing, eddies which can be resolved, and less remains to be modelled.

In some areas of Fluid Mechanics SGS modelling was very successful, even in the cases where some very sophisticated time-averaged turbulence closure techniques failed (for detailed list of references see Ferziger,1981). Unfortunately, the most of analysed cases are still far from the engineering applications. This research is primarily aimed to develop mathematical model and computer code based on LES, which will account for all specific characteristics of flow in lakes, and which will be suitable for engineering application. The equations are written in nondimensional form, and due to quite different length scales for horizontal and vertical directions, some statements of relative significance of some terms in equations are given.

MATHEMATICAL MODEL

Navier-Stokes' equations for horizontal directions with included effects of earth rotation [Iwasa & Inoue,1984] are filtered [Deardorf,1970,Ferziger,1981], and resulting equations for horizontal directions are:

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j) + \frac{\partial}{\partial x_j}(L_{ij}) = 2\varepsilon_{ijk}\bar{u}_j\Omega_k - \frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(C_{ij}) \tag{1}$$

$$L_{ij} = \overline{\bar{u}_i \bar{u}_j} - \bar{u}_i \bar{u}_j \qquad \textit{Leonard's terms}$$

$$C_{ij} = \overline{u'_i \bar{u}_j} + \overline{\bar{u}_i u'_j} + \overline{u'_i u'_j} \qquad \textit{Reynolds subgrid scale stress}$$

Leonard's terms can be written explicitely because they depend on the type of the filter used, and SGS stress terms should be modelled. Eddy viscosity approach and Smagorinsky's model based on local turbulence equilibrium are used to model SGS stress.

In this analysis space averaging is rather formal step used to explain physics of flow, not necessarely to give equations which are to be solved. Leonard's terms which make resulting equations odd are not discussed here. Temporarely accepting the sugestion of many authors [Baron,1982,Ferziger,1981] they are dropped, even though some calculations of them like ancillary results show that they might be of some influence. This remains to be investigated later.

More important is the way of defining of eddy viscosity. Smagorinsky model is:

$$C_{ij} = \nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) \tag{2}$$

$$\nu_t = (c\Lambda)^2 S = (c\Lambda)^2\left[\frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right]^{1/2} \tag{3}$$

where (c) is a constant estimated to be between 0.0675 and 0.23, and Λ characteristic grid spacing over which the equations are filtered. For this type of flow, control volumes are very flat, and grid spacing for vertical direction is usually two order of magnitude less then for horizontal. That allows us to state that Λ is of the order of D, rather then of L. In other papers [Moin & Kim,1982] it was some average of grid spacing in all directions. If U is reference velocity, L and D reference horizontal and vertical lengths respectively, equation for eddy viscosity can be written in dimensionless form, and with velocities u,v,w instead of u_i:

$$A_t\nu_t^+ = (c\Lambda)^2\frac{UD^2}{L}\left\{2\left(\frac{\partial u^+}{\partial x^+}\right)^2 + 2\left(\frac{\partial v^+}{\partial y^+}\right)^2 + \left(\frac{\partial u^+}{\partial y^+} + \frac{\partial v^+}{\partial x^+}\right)^2 + \frac{L^2}{D^2}\left[2\left(\frac{\partial w^+}{\partial z^+}\right)^2 + \left(\frac{\partial u^+}{\partial z^+}\right)^2 + \left(\frac{\partial v^+}{\partial z^+}\right)^2\right]\right\}^{1/2} \tag{4}$$

The group of terms in square brackets will be retained, and the rest will be dropped. Obviously ($c\Lambda$) is some sort of mixing length, so here the problem is the definition of mixing length (like in Radojkovic & Ivetic, 1982). For this example Λ was equal to vertical grid spacing. c was variable, but constant in one computer run. Equation (1) for x direction, with At reference viscosity, reads:

$$\frac{\partial u^+}{\partial t^+} + Re\left[\frac{\partial}{\partial x^+}(u^+u^+) + \frac{\partial}{\partial y^+}(u^+v^+)\right] + \frac{L}{D}Re\frac{\partial}{\partial z^+}(u^+w^+) = f^+v^+ - Re\frac{\partial p^+}{\partial x^+} + 2c^2\frac{D}{L}Re\frac{\partial}{\partial x^+}\left[\frac{\partial V_*^+}{\partial z^+}\frac{\partial u^+}{\partial x^+}\right] + c^2Re\frac{D}{L}\frac{\partial}{\partial y^+}\left[\frac{\partial V_*^+}{\partial z^+}\left(\frac{\partial u^+}{\partial y^+} + \frac{\partial v^+}{\partial x^+}\right)\right] + c^2Re\frac{L}{D}\frac{\partial}{\partial z^+}\left[\frac{\partial V_*^+}{\partial z^+}\left(\frac{\partial u^+}{\partial z^+} + \frac{D}{L}\frac{\partial w^+}{\partial x^+}\right)\right] \tag{5}$$

where $\partial V_*^+/\partial z^+$ stands for terms in square brackets in the equation (4), and $Re=UL/A_t$ some form of Reynolds number. In the equation (5) the importance of the last term on the left hand side is at least the same as of the other convective terms. The most of the external influences (wind and bottom shear stresses) are introduced through the last term on the right hand side which has to be modelled the most carefully.

RESULTS

Numerical method used in this research differs slightly from that explained earlier (Iwasa & Inoue,1984). Scheme is explicit staggered in space and in time (leap-frog), and momentum equations for x and y directions are not integrated over the same control volume. Simple test reservoir is chosen, square in horizontal plan (800 m x 800 m), with uniform depth (=16m). Water is set into motion by uniform shear stress applied over free surface. Results of computations are used only to illustrate some properties of this approach. Time step was equal to 4 s what was the maximum for solving of surface gravity waves. Period of gravity waves was approximately 128 s, as can be seen in the figs.1. & 2. where time histories of water level changes in a corner of the reservoir, surface and bottom velocities (for central column) are plotted. The surface waves are in the class of the fastest changes in the lake, and if one wants to resolve them, time step, short like this, has to be used. The other type of motions (that are usually more important for mass transport) are visible in the figs.1. & 2. : surface velocity plot shows response of lake to uniform shear flow, and bottom velocity plot shows flow instability in lower layers caused by the presence of bottom and downwelling motions from above. Both motions are of much longer period, and for their solution biger time step should be adopted. In figs. 1., 3. wind (and surface shear stress) has south to north direction and in Figs. 2., 4. southwest to northeast.Wind speed was 10 m/s. Reservoir boundaries are paralel to coordinate axis.

CONCLUDING REMARKS AND OBJECTIVES FOR FURTHER RESEARCH

At this stage all expected benefits of this type of modelling are not yet obvious, but encouraging is the fact that computation time is not considerably increased when such sophysticated model is introduced. Dimensional analysis shows that the most important is vertical momentum exchange between layers, but in some parts of flow domain, horizontal acceleration is also important, and cannot be neglacted. If Smagorinsky's model for local equilibrium is used the problem is shifted to mixing length matching. Mixing length has a role of filtering, too big one will damp all, otherwise resolvable, large eddy motions.

Before applying this model on real lakes and reservoirs, our efforts will be directed in the following:

- improving of the vertical momentum transfer modelling, especially near the bottom (refined grid) and free surface;
- investigating the role of Leonard's terms on momentum transfer;
- investigating the influence of thermal stratification on vertical momentum transfer(through mixing length estimates);
- and, if necessary, introducing semi implicit numerical scheme (to enable longer time steps).

ACKNOWLEDGEMENT

The present work was done during the second author's stay at Kyoto University, Laboratory for River Engineering, as student on International Course with Monbusho scholarship. The computations were performed at the Data Processing Center of Kyoto University on computer FACOM M-382.

SELECTED REFERENCES

[1.] F.Baron,1982,Macro-simulation tridimensionnelle d'ecoulements turbulents cisaibles, Rapport EDF - DER HE41/82-21 HE 44/82-20

[2.] J.W.Deardorf,1970,A Numerical Study of three-dimensional turbulent channel flow at large Reynolds numbers,J.Fluid Mech.,vol.41.,part 2,pp.453-462.

[3.] J.H.Ferziger,1981,Higher-level Simulations of Turbulent Flows, Technical Report Number TF-16,

[4.] Y.Iwasa,K.Inoue,1984,Plane multi-layred (or three-dimensional) flow analysis for lake Biwa, HYDROSOFT'84,ed.C.A.Brebbia,C.Maksimovic,M.Radojkovic,Elsevier,

[5.] P.Moin and J.Kim,1982,Numerical investigation of turbulent channel flow,J.Fluid Mech.,vol.118,pp.341-377,

[6.] M.Radojkovic,M.Ivetic,1982,Finite Elements Analysis of momentum transfer in rivers of complex cross section, Finite Elements in Water Resources, Springer-verlag,

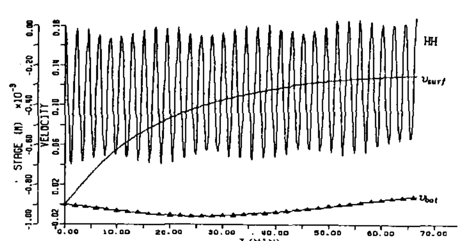

Fig.1. Time histories of water level (HH), surface velocity (u_{surf}) and bottom veloc ty (u_{bot}) due to south-north wind (speed=10 m/s).

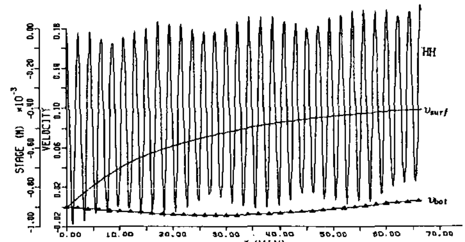

Fig.2. Time histories of water level (HH), surface velocity (u_{surf}) and bottom velocity (u_{bot}) due to southwest-northeast wind (speed=10 m/s).

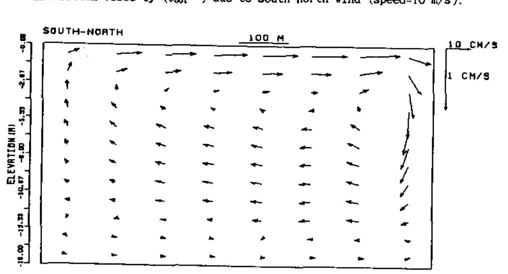

Fig.3. Velocities in the vertical cross section through the center after 1000 time steps, due to south-north wind (speed=10 m/s).

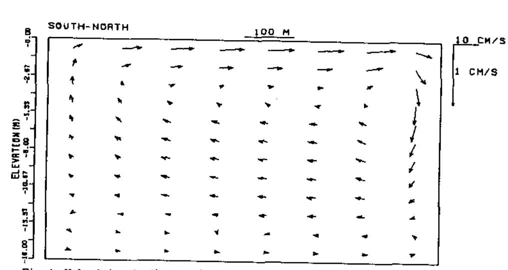

Fig.4. Velocities in the vertical cross section through the center of the reservoir after 1000 time steps due to southwest-northeast wind (speed=10 m/s).