II-84 PARAMETER IDENTIFICATION FOR SIMULATION OF INFILTRATION PROCESS

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1. INTRODUCTION

The solution of Richard's equation requires a prior knowledge of soil moisture characteristics in the form of moisture-suction ($\theta - \psi$) and conductivity-suction ($k - \psi$) relationships, and various models are used to represent these relations. As the soil properties as well as th $\theta - \psi - k$ inter-relations vary considerably from soil to soil it is necessary to identify proper models and estimate their parameters.

2. $\theta - \psi$ RELATIONSHIP

Moisture-suction data obtained from 24 Kanto-Loam soil samples taken at depths ranging from 60-600 cms. were tested against four models for their applicability. The parameters in each model were computed by an optimisation technique based on sensitivity analysis. The models considered were

1) $\theta = (\theta_o - \theta_r)\alpha / \{\alpha + (\ln \psi)^{\beta}\} + \theta_r$

2) $\theta = (\theta_o - \theta_r)\exp\{\alpha(\psi_{cr} - \psi) + \theta_r$

3) $\theta = (\theta_o - \theta_r)\ln(\psi - \psi_{cr} + 1) / \ln(\psi_r - \psi_{cr} + 1)$

4) $\psi = \psi_{cr}\{\theta / \theta_o\}^{-b}$

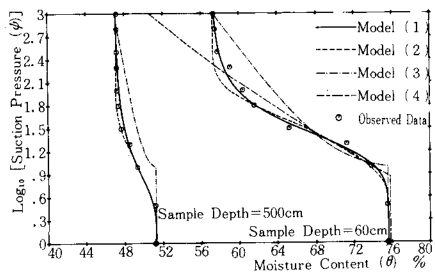

Fig. 1.

The model 1) was found to yield the best agreement with the observed data as shown in the fig. 1.

3. $k - \psi$ RELATION

Unlike the $\theta - \psi$ data, $k - \psi$ data cannot be easily obtained from the laboratory sample tests. The experimental data appears to be sensitive to the method employed and sampling. Therefore $k - \psi$ relation is decided beforehand and the saturated conductivity is identified by optimising the parameter in a fully implicit one-dimensional numerical model of Richard's equation by comparing with double-ring field infiltration test data.

$k - \psi$ relationship is represented by the model $k = k_o\, Se^n$

where $Se = (\theta - \theta_r)/(\theta_o - \theta_r)$: $n = .015\, w + 3.0$ and $w = \int_0^{\psi_r} \gamma\, \psi\, d\theta$; $\psi_r = 15$ atm.

For the estimation of k_o , let $\hat{Q}$ be the infiltration computed by the numerical model for one-dimensional Richard's equation at time t for an arbitrary initial estimate of k_o and Q be the observed infiltration rate.

Infiltration rate $\hat{Q} = \{k(\partial\psi/\partial z - 1)\}^t = \Phi(k_o)$

If $k_o + \Delta k_o$ is the true parameter value for $\hat{Q} \rightarrow Q$

then
$$Q = \Phi(k_o + \Delta k_o) = \Phi(k_o) + \frac{\partial \Phi}{\partial k_o} . \Delta k_o$$

$$Q - \hat{Q} = Se^n \{(\partial\psi/\partial z) - 1\} . \Delta k_o \quad .... (1)$$

This eqation can be written for many data points so that a matrix equation results, which can be solved by regression techniques for Δk_o. Taking the new parameter as $k_o + \Delta k_o$ the iteration procedure is continued until Δk_o becomes negligibly small.

The algorithm was validated by a numerical example shownin fig. 2. By taking simulated results for $k_o = .001$ as observed data k_o was computed with initial estimates of k_o = .01 and k_o = .0001. The computed k_o value converged to the true value within four iteration in each case.

Field infiltration data were obtained using double ring infiltrometer with inner cylinder diameter 10 cms. and outer cylinder diameter 50 cms. Saturated hydraulic conductivity was computed using the above method and an example of the results is shown in fig 3 .

4. RESULTS AND DISCUSSION

Model 1) was identified to as the best fit model for the moisture-suction relationship for the Kanto-Loam soil. The parameters involved could be easily computed by the optimization technique using about 10 data points. The saturated hydraulic conductivity can be estimated by analysing the double-ring infiltration test data using a numerical model. The method described is stable and converges rapidly to the optimum parameter value. Field variability of k_o can be identified by performing several tests scattered over the area.

The parameter n in the $k - \psi$ relation too can be identified by the same manner, or else the method can be extended to identify parameters if a different model is selected for the representation of $k - \psi$ curve. In that case equation (1) becomes

$$Q - \hat{Q} = \frac{\partial \hat{Q}}{\partial P_i} . \Delta P_i \qquad i=1,m; \; P_i = \text{ith parameter}; \; m = \text{number of parameters}$$

$\partial \hat{Q}/\partial P_i$ terms can be computed by differentiating $\hat{Q} = \int_o^{zn} \frac{\partial \theta}{\partial t} dz - k_{zn}$ to give

$$\frac{\partial \hat{Q}}{\partial P_i} = \int_o^{zn} c(\psi) \frac{\partial \xi}{\partial t} dz - \frac{\partial k}{\partial P_i} zn \quad \text{where } \xi = \frac{\partial \psi}{\partial P_i} \quad \text{and } c(\psi) = \frac{\partial \theta}{\partial \psi}$$

The sensitivity coefficients ξ can be computed by differentiating the governing equation with respect to each parameter and solving together with the governing equation.

Sensitivity equations take the form $c(\psi)\frac{\partial \xi}{\partial t} = \frac{\partial}{\partial z}\left\{ \frac{\partial k}{\partial P_i}\left(\frac{\partial \psi}{\partial z} - 1 \right) + k \frac{\partial \xi}{\partial z} \right\}$

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1) Decoursey D. G and W. M. Snyder (1969) Computer Oriented Method of Optimising Hydrological Model Parameters, Jrnl of Hydrology 9, pp 34-56

2) Havercamp, R., M. Vaclin, J. Touma, P. J. Wierenga, and G.Vauchad, G. (1977) A Comparison of Numerical Simulation Models For One-Dimensional Infiltration, Soil Sci. Soc. Am. J., Vol 41, pp 285-293

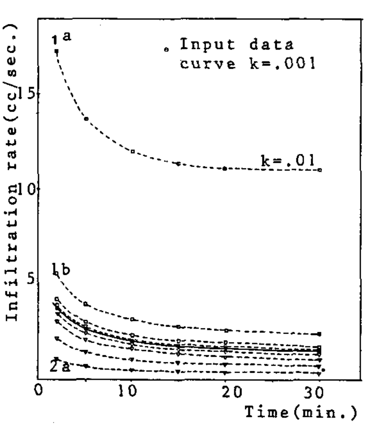

Fig. 2. Validation of Optimisation Algorithm.

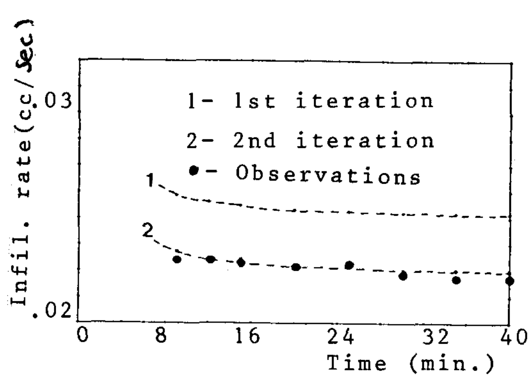

Fig.3 Computation of k from Field data.