I-549 DISPLACEMENT-BASED FINITE ELEMENTS FOR THE DAM-RESERVOIR SYSTEM

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1. Introduction

The system of dam-reservoir is usually discretized and analysed using an Euler-Lagrangian FEM formulation, i.e.; the dam nodal parameters are displacements whereas those of the reservoir are pressures. Although the method is effective, but involves asymmetric and very large banded coefficient matrices when the system of equations is to be solved, so very special treatments and algorithms are needed for the calculation of response or the non-standard eigen problem. Here a full Lagrangian formulation is adopted for both fluid and solid domains with which the standard FEM programs could be employed ,and the interaction is naturally considered while the interface nodes have 2 independent set of tangential motions to admit smooth slipping over there. This could be guaranteed by interface elements or other methods. In this work curved zero thickness isoparametric elements are used for this purpose. Energy radiation into infinity is approximated by the Sommerfeld condition. At last the effectiveness of such formulation is inspected for a vertical flat rigid dam or tentatively for a flexible arch dam.

2. Theory

Dam could be discretized just by standard Lagrangian FEM to give its contribution to the property matrices. The reservoir could be discretized in the same manner[i] if one notes that under seismic transient shocks the fluid motion could be assumed small, compressible, and invicid. Besides, for stability considerations an irrotational constraint is applied using penalty function technics. The main and constraint constitutive relations for the fluid media are expressed as $P = k\ .\mathrm{div}U$,and $\{\tau\} = [D^*]\mathrm{curl}U$ respectively, where U is displacement, k is the bulk elastic modulus and $[D^*]$ is a diagonal rotationall elasticity matrice with very large elements. Variation of the potential energy of such fluid element yields;
$[K_F] = \int_V [B]^T [D] [B] dV$ as the stiffness matrice (which should be computed by the reduced integration technics). [B] is the matrice obtained by the product of the strain operator and the shape function [H], and [D] is the diagonal matrice of the total elastic properties defined above. The shape function H should be from the Lagrange family due to the penalty function considerations. Morever it is desirable to have an account for the gravity surface waves, for which the linear wave theory was employed to augment the stiffness by $[S] = \int_A \{h\}^T w \{h\} dA$, where $\{h\}^T$ is the one row of [H] corresponding to vertical direction and w is the unit weight of water. The 3D interface[ii] element has an infinite (large) normal and two zero tangential stiffnesses, thus reflecting the actual phenomenon between water and the dam. The only contribution of this element is an stiffness part. The global property matrices [M], [C], and [K] are assembled over the three element types to give the equation of motion as;
$[M]\{\ddot{a}\} + [C]\{\dot{a}\} + [K]\{a\} = -[M]\{\ddot{a}_g\}$ with {a} as the nodal displacements. As for the boundary conditions, the free surface was treated similar to an elastic foundation as discussed above but the infinite media should be represented by a special B.C. admitting the energy propagation into the infinity. According to the assumption that at far enough distances from the dam the wave front might have a planar shape normal to the progression direction, Sommerfeld radiation condition;
$dU_n/dn = -\dot{U}_n/c$ was adopted for the reservoir truncated boundary. Morever for the reservoir bank a partial radition modified by the relative impedance $\beta = \gamma V_P / c\, \gamma_w$ was used as its significance is noted by many authors. The latter B.C. is expressed as;
$dU_n/dn = -\dot{U}_n/(\beta c)$. Here n is the boundary normal direction and U_n the normal displacement. c and V_P are the compressional wave velocities in water and in the bank material and γ_w or γ the mass density of each. Note that the damping matrice thus formed, is nonproportional and should be augmented by any internal damping.

3. Numerical Results

Both solid and fluid elements were taken as 3D 8-noded isoparametrics while the interface element was 4-noded. The integration order used was equal to one for the fluid element but equal to two for others. The rotational elastic moduli was assumed as high as 100 times the bulk moduli ($k=211510\ t/m^2$). A vertical 3.^{m} thick, 60.^{m} high and 60.^{m} long cylindrical dam was solved against a reservoir of length 2H. Despite the coarseness and non-compatibility of the two meshes the static case (i.e; under the gravity action) indicated results well in agreement with the case in which the hydrostatic load is applied as a surface traction. The modal extraction of a simple rectangular reservoir of 5.08×1.905^m size was also excecuted. The spectrum is composed of three distinguished parts; zero energy, sloshing and impulsive modes. The 1st sloshing,and the 1st impulsive modes were in agreement with their analytical values neglecting the 3.% and 1.% errors respectively. Furthermore the maximum steady state hydrodynamic pressure on a flat rigid wall of 60.^{m} height and 60.^{m} length under simple harmonic horizontal motion of the wall was obtained using this formulation. Good agreement with the analytical solution was concluded when the excitation frequency N was lower than the 1st natural frequency of the reservoir N^R. In this case the radiation B.C. does not control the result in contrast with the case $N > N^R$ in which the solution is very sensitive to the B.C. and it essentially should admit the radiation.

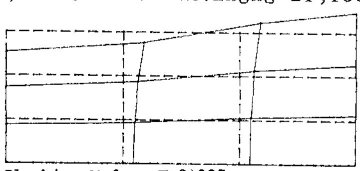
4. Conclusions

The new method of FEM formulation for the fluid-structure interaction works promis ingly well. Its merit is its relatively simple and smart algorithm to solve the complicated dam-reservoir system. It could be implemented by adding one or two new types of elements to the element library of the existing FEM programs. For the response analysis step-by-step integration is recommended.

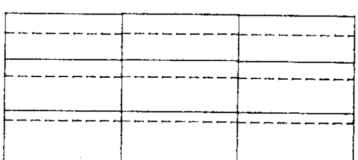
5. References

i) E.L.Wilson et al,"Finite Elements for The Dynamic Analysis of Fluid-Structure Systems",Int.J.Num.meth.Engng 19,1983

ii) G.Beer,"An Isoparametric Joint/Interface Element For Finite Element Analysis",Int.J.Num.meth.Engng 21,1985

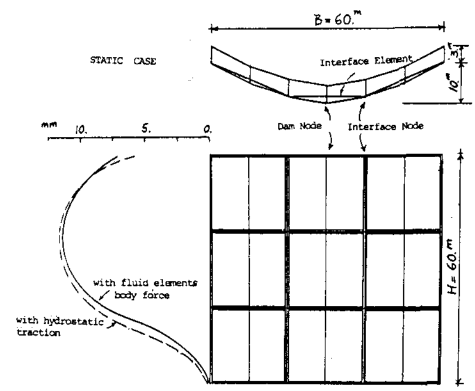


1-st Sloshing Mode T=2.925 sec.
(Analytical T=2.808 sec.)



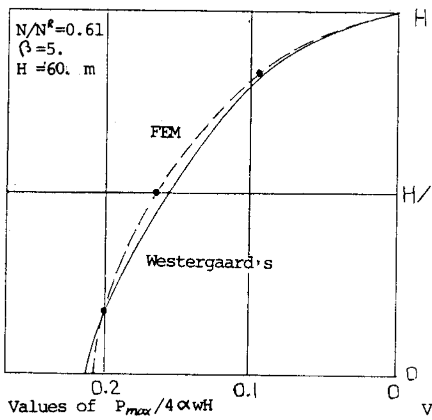
1-st Impulsive Mode T=0.00535 sec.
(Analytical T=0.00529 sec.)

Natural Modes of 5.08 x 1.905 m Reservoir

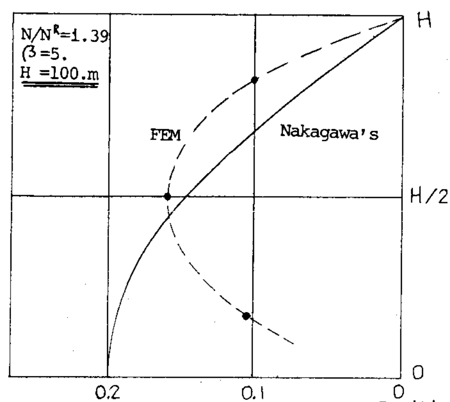


Crown Cantilever Downstream Displacement

SIMPLE ARCH DAM/INTERFACE FEM MODEL



Values of $P_{max}/4\alpha wH$



Values of P/4αwH At Downstream Extereme Position

(P =Hydrodynamic Pressure On The Rigid Wall , α =Excitation Coef.)