I-60 AN ELASTIC POST-BUCKLING BEHAVIOUR OF PROPPED-CANTILEVER COLUMN

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1. INTRODUCTION

It is well-known through the elliptical integrals[1] or the non-linear FEM analysis that an elastic straight cantilever column of uniform section is always stable after buckling, and the load can monotonically increase even when the displacement becomes very large, as shown in Fig. 1. Naturally, one might expect that the same column but with other boundary conditions shows similar deformational characteristics as the cantilever column. However, as can be seen later, it is not true for the case of the column with one end fixed and the other hinged, as called the propped-cantilever column. This paper is intended to present the result of an elastic finite displacement analysis of the straight propped-cantilever column with uniform section and to discuss its characteristics in comparison with the cantilever column case.

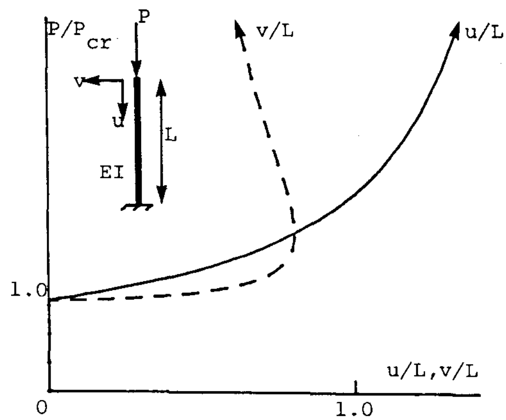

Fig.1 Post-buckling behaviour of cantilever column

2. NUMERICAL RESULTS

A propped-cantilever column illustrated in Fig. 2 is analysed by the FEM updated Lagrangian formulation of plane beam element. In order to make sure the accuracy, the number of elements adopted is 32, and the standard Newton-Raphson iteration scheme is employed with the required tolerance of convergence check setting to be 10^{-3}. It is noted that a disturbing moment of the small magnitude of $P_{cr}L/1000$ ($P_{cr}=20.19EI/L^2$=an elastic buckling load of the column) is applied at the top to avoid bifurcation. By using the non-dimensionlized quantities of load P/P_{cr}, vertical displacement u/L at the top, slope angle θ (in radian) at the top, and horizontal displacement v/L at the middle of the column, the load-displacement curves of the propped-cantilever column are plotted in Fig. 3. Fig. 4 illustrates the deformed configurations of the column at the various equilibrium states corresponding to the encircled points indicated in Fig. 3.

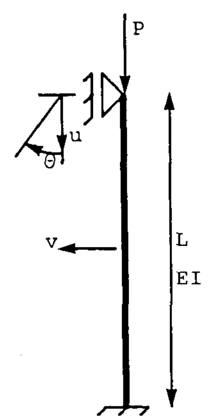

Fig.2 Propped-cantilever column

3. DISCUSSIONS

It is very interesting to note that there exists an unstable region between points ② and ④ which is a significant difference from the well-known case of cantilever column. Observing the deformed configurations of Fig. 4, the unstable behaviour may result from the fact that, because of this particular boundary condition, the deformation exhibits the double curvature shape from the

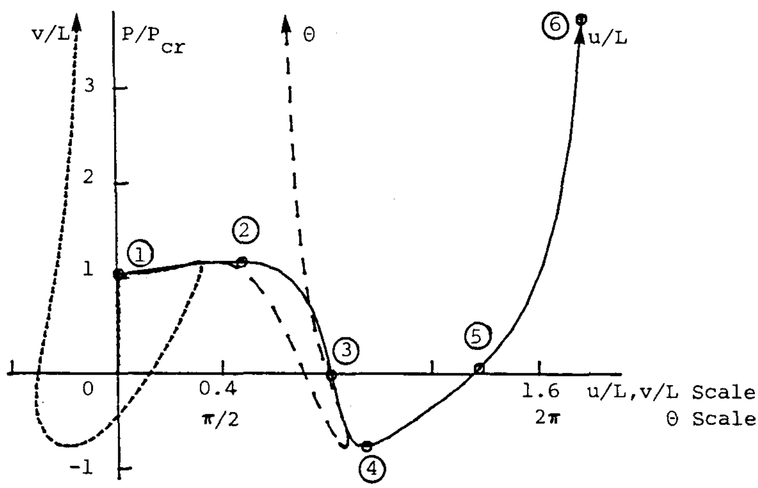

Fig.3 Post-buckling behaviour of propped-cantilever column

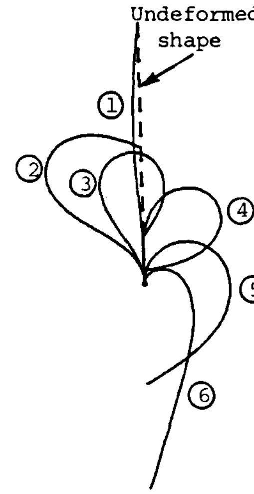

Fig.4 Deformed configuration

beginning to the state ④. For the double curvature equilibrium shape, as illustrated in Fig. 5, there always exists the inflection point of curvature $\kappa=0$ at some location and then resisting bending moment being zero. While the well-known cantilever column can increase the load through the monotonically increasing single curvature, that is, the monotonically increasing bending moment, the unstable behaviour of the propped-cantilever column can well be explained by such double curvature deformational characteristics. Whether the double curvature shape exhibits stable (beginning to point ②) or unstable (points ② to ④) can be examined by taking the free-body equilibrium of portion 1) ($\kappa>0$) in Fig. 5. It is also noted that the equilibrium is stable and the load can increase monotonically after point ④ due to the continuously increasing single curvature.

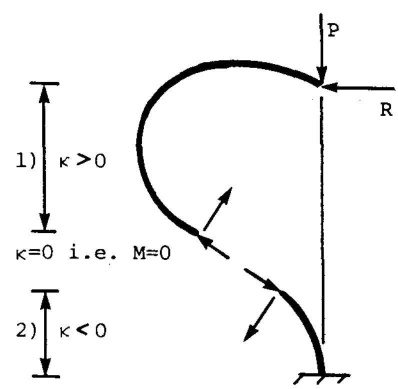

Fig.5 Double curvature deformation

4. CONCLUDING REMARKS

An investigation of an elastic post-buckling behaviour of the straight propped-cantilever column of uniform section is presented. The numerical result shows very interesting deformational characteristics, which are significantly different from the well-known case of cantilever column. Theoretically, the propped-cantilever column is expected to be solved by a contact problem of the elliptical integrals, where the domain is separated into two parts of $\kappa>0$ and $\kappa<0$, each of which is the single curvature problem similar to the cantilever column case but with unknown boundary position of inflection point.

Reference 1) Timoshenko, S.P. and Gere J.M., THEORY OF ELASTIC STABILITY, Second Edition, McGraw-Hill, 1961.