# I-56 NON-ITERATIVE EFFICIENT NONLINEAR ANALYSIS FOR SPACE STRUCTURES

Kawasaki Heavy Industries Ltd., K. K. Liyanage
Univ. of Tokyo, Member, Akio Hasegawa
Univ. of Tokyo, Member, Fumio Nishino

**1. INTRODUCTION:** The common practice at present in analysing the non-linear load-displacement behaviour of beams and frames is to adopt iterative schemes such as Newton-Raphson technique. However, it is rather inconvenient to employ such procedures in practice owing to the fact that the the checking of convergence need be carried out at each incremental step. Intead, the present study is to establish a non-iterative solution scheme to obtain the non-linear load-displacement behaviour of elastic spatial beams and frames with general initial configurarion, boundary and loading conditions.

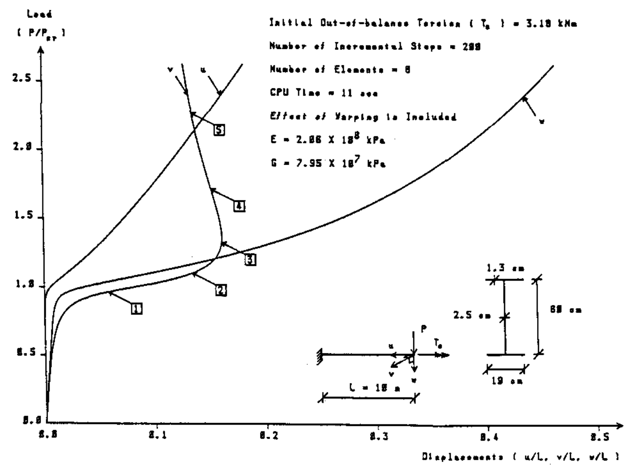


Fig.1 - Load-Deflection Behaviour of a Cantilever Beam

**2. TRANSFORMATIONS AND ASSEMBLING:**

The incremental stiffness equation for a straight thin-walled beam element which is in equilibrium at an arbitrary reference state, presented in Ref.1, is utilized herein to obtain the load-displacement behaviour of plane and spatial members and frames, as

$$\mathbf{F} = \mathbf{K}_T\,(\mathbf{P}^0)\,\mathbf{d}$$

in which $\mathbf{F}$ and $\mathbf{d}$ are the incremental load and displacement vectors including warping components, and $\mathbf{K}_T$ is the tangent stiffness matrix in terms of the stress resultants ( $\mathbf{P}^0$ ) present at the reference state. Before the assembling process, three transformations are needed. Firstly, the load and displacement vectors are rearranged separated to those corresponding to the two end nodes in vector representation. Secondly, all the components are referred to a single point on the cross-section, for example, the centroid as in this study. Finally, the load and displacement vectors are transformed from the element local coordinates to global coordinates. After performing the above transformations, the global stiffness equation is assembled for the whole structure and solved for the next incremental step, utilizing the path length control technique.

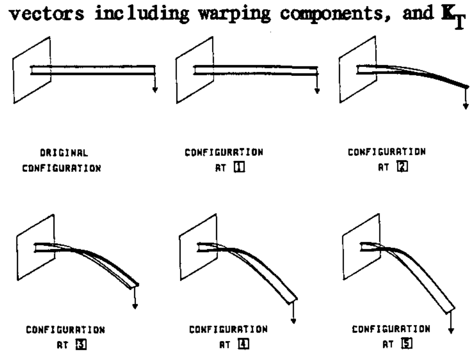


Fig.2 - Deformed Configurations of a Cantilever Beam

**3. UPDATING PROCEDURE:**

(1) Updating of Coordinates: The element end coordinates are first updated by simply adding the incremental displacements in

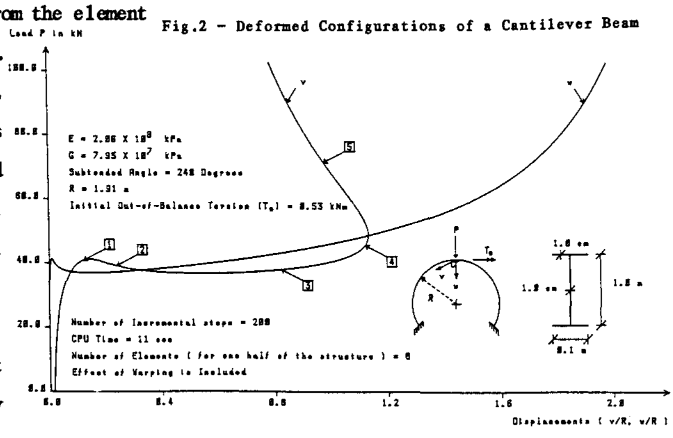


Fig.3 - Load-Deflection Behaviour of a Fixed Circular Arch

globalcoordinates to the current coordinates. Next, the increments in element end rotations in local coordinates are added to the current values and thus the new element end directions and the new local coordinate axes are found.

(2) Updating of Stress Resultants: The increments in the element end forces are found by re-substitution of the incremental displacements to the individual element stiffness equations, and hence the new element end forces can be found in the global coordinates. The element end forces are then transformed to the element local coordinates, and the stress resultants for the new reference state are found.

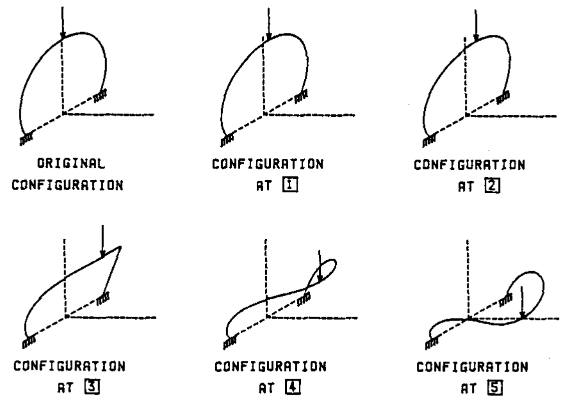


Fig.4 - Deformed Configurations of a Fixed Circular Arch

4. NUMERICAL EXAMPLES:

(1) Considered first is the post lateral-buckling behaviour of a uniform cantilever beam with doubly symmetric I-section under a vertical load at the free end. The load-displacement relations and the deformed configurations are shown in Figs.1 and 2, respectively.

(2) The post lateral-buckling load-displacement behaviour of a uniform circular fixed arch with doubly symmetric I-section under a vertical load at the crown is investigated next. The load-displacement relations are shown in Fig.3 and the corresponding deformed configurations are also given in Fig.4.

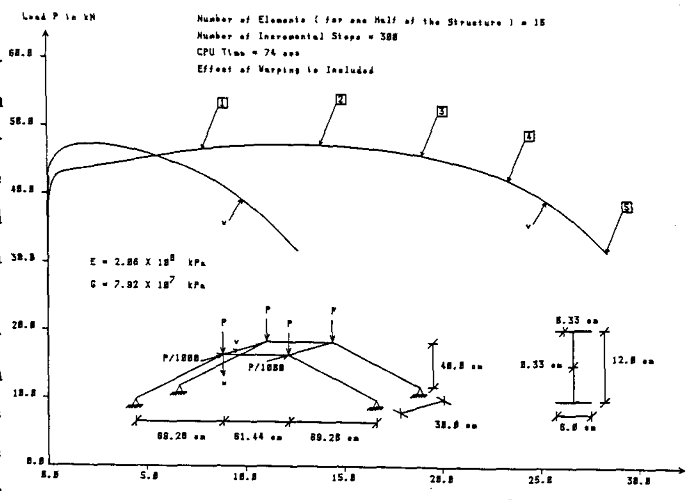

Fig.5 - Load-Deflection Behaviour of a Space Frame

(3) Considered finally is a space frame with doubly symmetric I-section, as shown in Fig.5. The load-displacement behaviour and the deformed configurations are shown in Figs.5 and 6, respectively.

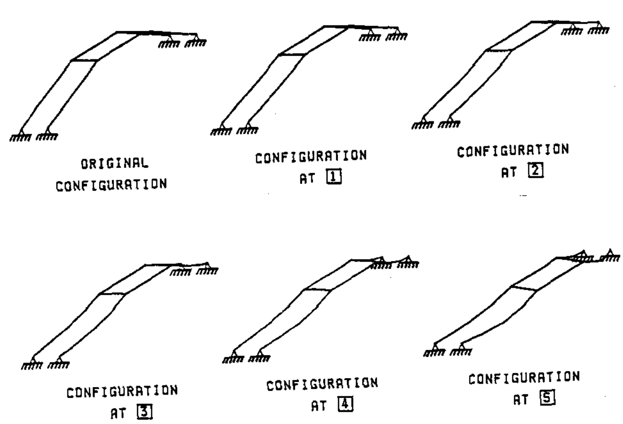


Fig.6 - Deformed Configurations of a Space Frame

5. CONCLUSIONS: The non-linear load-displacement behaviour of spatial structures has been obtained with sufficient accuracy by the direct solution of incremental stiffness equation. This procedure makes unnecessary to iterate or to check convergence throughout, and thus reduces a lot of computational effort.

6. REFERENCES: (1) Hasegawa, A et al., 'A Concise and Explicit Formulation of Out-of-Plane Instability of Thin-Walled Members',Structural Engineering/Earthquake Engineering, Vol.2,No.1, April 1985, Proc. of JSCE.