I-31 **Shakedown Analysis of Multi-Span Beams with Consideration of Bending and Shear**

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1. Introduction In shakedown analyses, the method of nonlinear programming is generally used. However, in the case of two-span beams or symmetric three-span beams subjected to a single repeated moving load, it may be much more easily treated as a problem of quadratic programming, for the reason that only two variables of load multiplier and residual reactive force are reduced in the mathematical programming.

2. Shakedown Analysis Figure 1 shows the beam model which are divided into n discrete elements. Here we start first on inducing the expressions for three-span beams. The elastic bending moment and shearing force at some location, say i, under a repeated moving load, being denoted by $M_i(\xi)$ and $S_i(\xi)$, an applied elastic stress domain in m_i-s_i plane is obtained as follows:

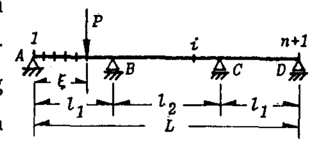

Fig. 1 Three-span beam

$$R_i = \left\{ m_i, s_i \;\middle|\; |m_i| \le \left| \frac{M_i(\xi)}{M_p} \right| ,\; |s_i| \le \left| \frac{S_i(\xi)}{S_p} \right| ,\; 0 \le \xi \le L \right\} \tag{1}$$

which is graphically surrounded by the region with slashes shown in Fig. 2 for demonstration. M_p, S_p are fully plastic moment and shearing force. Hereby, a circumscribed polyhedron $\bar{R}_i$ about R_i may be defined so as to be convex, and possesses some vertexes (m_{ij}, s_{ij}), marked by dots in figure.

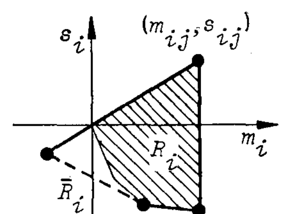

Fig. 2 Vertexes and convex region

We may assume rationally that the residual forces at two intermediate supports will be the same after some repetition of applying the moving load along the beam, and give them X, from which the residual bending moment and shearing force at each view point are presented as $\bar{M}_{i1}$, $\bar{S}_{i1}$ and $\bar{M}_{i2}$, $\bar{S}_{i2}$ with respect to intermediate supports B and C, respectively, and are nondimensionlized into $\bar{m}_{ij}=\bar{M}_{ij}/M_p$, $\bar{s}_{ij}=\bar{S}_{ij}/S_p$ (j=1,2). Further, introducing a load multiplier $\lambda=Pl_1/M_p$ and a nondimensional variable $\mu=Xl_1/M_p$, determination of the shakedown load factor λ_s from Melan's theorem yields the following mathematical programming:

$$\lambda_s = \text{maximize } \lambda \quad \text{subject to} \quad [\lambda m_{ij}+\mu(\bar{m}_{i1}+\bar{m}_{i2})]^2+[\lambda s_{ij}+\mu(\bar{s}_{i1}+\bar{s}_{i2})]^2 \le 1 \tag{2}$$

The inequality constraints in problem (2) may be developed into a quadratic one about μ as

$$[(\bar{m}_{i1}+\bar{m}_{i2})^2+(\bar{s}_{i1}+\bar{s}_{i2})^2]\mu^2+2[m_{ij}(\bar{m}_{i1}+\bar{m}_{i2})+s_{ij}(\bar{s}_{i1}+\bar{s}_{i2})]\lambda\mu+[(m_{ij}^2+s_{ij}^2)\lambda^2-1] \le 0 \tag{3}$$

from which a necessary condition on real solution of - yields

$$\lambda \le \sqrt{(\bar{m}_{i1}+\bar{m}_{i2})^2+(\bar{s}_{i1}+\bar{s}_{i2})^2} \Big/ m_{ij}(\bar{s}_{i1}+\bar{s}_{i2})-s_{ij}(\bar{m}_{i1}+\bar{m}_{i2}) \tag{4}$$

While inequality (3) gives a constraint about μ as

$$\gamma_{ij}^{-} \leq \mu \leq \gamma_{ij}^{+} \qquad \begin{matrix}\gamma_{ij}^{-}\\ \gamma_{ij}^{+}\end{matrix} = \frac{1}{(\bar{m}_{i1}+\bar{m}_{i2})^2+(\bar{s}_{i1}+\bar{s}_{i2})^2} \left[\begin{matrix} -[m_{ij}(\bar{m}_{i1}+\bar{m}_{i2})+s_{ij}(\bar{s}_{i1}+\bar{s}_{i2})]\lambda \\ \pm\sqrt{(\bar{m}_{i1}+\bar{m}_{i2})^2+(\bar{s}_{i1}+\bar{s}_{i2})^2-[m_{ij}(\bar{s}_{i1}+\bar{s}_{i2})-s_{ij}(\bar{m}_{i1}+\bar{m}_{i2})]^2\lambda^2} \end{matrix} \right] \tag{5}$$

Consequently, shakedown load factor λ_s satisfying inequalities (4) and (5) can be solved by applying following expression

$$\mu_s = \underset{i,j}{\text{maximize}}\ \gamma_{ij}^{-} = \underset{i,j}{\text{minimize}}\ \gamma_{ij}^{+} \tag{6}$$

in an iterative procedure within the region

$$0 \leq \lambda_s \leq \lambda_o \ , \ \lambda_o = \underset{i,j}{\text{minimize}} \left\{ \sqrt{(\bar{m}_{i1}+\bar{m}_{i2})^2+(\bar{s}_{i1}+\bar{s}_{i2})^2} \Big/ |m_{ij}(\bar{s}_{i1}+\bar{s}_{i2})-s_{ij}(\bar{m}_{i1}+\bar{m}_{i2})| \right\} \tag{7}$$

The procedure is graphically demonstrated in Fig. 3, where λ_e and λ_s means elastic limit load factor and shakedown load factor, respectively, while μ_s represents the residual reactive force factor at intermediate supports when the beam shakes down.

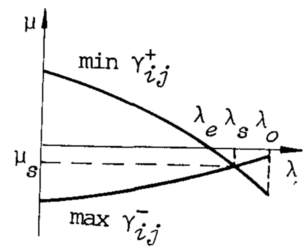

Fig. 3 Iterative procedure

As regard to the formulation for two-span beams, we drop out the terms $\bar{m}_{i2}$ and $\bar{s}_{i2}$ from Eq. (2) through (7) to formulate the corresponding ones and apply the above description.

3. Results Shakedown load factor vs. $M_p/(S_p l_1)$ is illustrated in Fig. 4. The dotted curved line indicates the variation of λ_s for two-equal-span beam while the curve right above it indicates variation of λ_s for three-equal-span beam. It can be seen that, at same value of $M_p/(S_p l_1)$ shakedown load factor of three-span beam is a little larger than that of two-span beam for three-span beam is stiffer than two-span beam under such circumstances. The other four curves below are for three-span beams with l_2/l_1 =1.5, 1.8, 2.2 and 2.5, respectively. What we may understand from the figure is that shakedown load factor fluctuates comparatively significantly only over certain region of ratio $M_p/(S_p l_1)$ for each case, and beyond that region shakedown load factor tends to a asymptotic value.

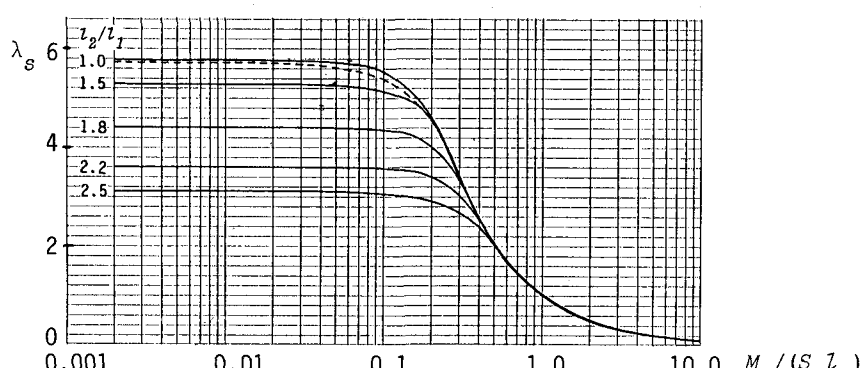

Fig. 4 λ_s vs. $M_p/(S_p l_1)$ (---:two-equal-span-beam,——:three-span-beam)