 ANALYTICAL MODELS OF BEAM-COLUMN

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1. Assumed Deflection Method

The deflected shape of an elastic-plastic beam-column is generally quite complex and requires numerical procedure for a solution. However, if we assume the deflected shape of a column to be a certain function and this general function or shape is assumed not to alter during further loading but merely changes its magnitude as the axial load increases, then, the beam-column problem is simplified drastically to a one-degree-of-freedom problem.

In this approach, we need to consider the equilibrium between external loads and internal resistance of a member only at one critical section of the column. For a symmetrically loaded column, this critical section is at mid-length of the column. The assumed deflection method is found to be most efficient for parametric studies and analytical modeling of the behavior and strength of beam-column problems among many available methods.

2. Deflection Functions

It is obvious that a proper choice of deflection shape is one of the key factors in this analysis. The assumed deflection function should be as close to actual deflected shape as possible.

The polynomial function is the type of deflection shape for an elastic beam subjected to lateral loads or end moments. Since sinusoidal function is the exact shape for an axially loaded column, it gives the exact solution for an axially loaded beam-column in the elastic range. This function is chosen for elastic as well as elastic-plastic analysis of beam-column here.

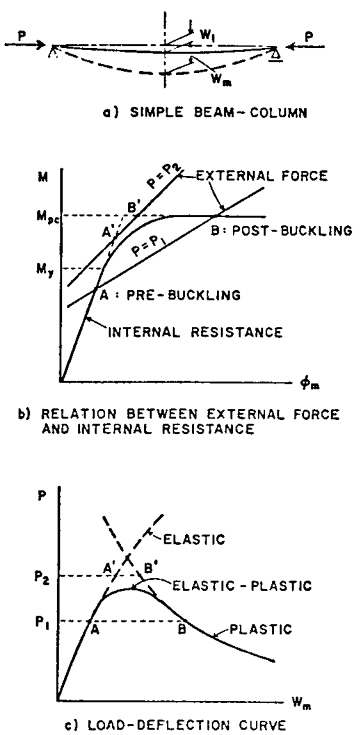

Figure 1.
General behavior of a Beam-Column.

3. Elastic Analysis (Before Buckling)

The bending moment at mid-length of the beam-column in Fig. 1a has the linear form:

$$M_{ext} = P\ (w_i + w_m) \qquad (1)$$

A relation between M_{ext} and the corresponding curvature Φ_m at mid-length is also linear in the elastic range as shown in Fig. 1b.

The internal resistance of a member follows the M-P-Φ curve. This nonlinear relation, though often assumed by an elastic-perfectly plastic relation, is also shown in Fig. 1b. In general, there are two intersecting points A and B and they correspond to two points in P-w relation (Fig. 1c); one in the pre-buckling and the other in the post-buckling branch as shown in Fig. 1c.

Taking the equilibrium condition at mid-length between the external and internal moments, the deflection due to axial load can be solved as follows (1):

$$(w_m)_{EL} = \frac{w_i}{\frac{P_{cr}}{P} - 1} \qquad (2) \qquad \begin{array}{l} w_i = \text{initial deflection} \\ P_{cr} = \text{Euler Buckling Load} \end{array}$$

4. Inelastic Analysis (Post-Buckling)

For an elastic-perfectly plastic M-P-Φ relation, the maximum internal bending moment at the critical section is always equal to the full plastic moment M_{pc}. The moment induced by external loads at the critical section is not affected by the particular type of assumed deflected shape in this range. The load-deflection equation can be expressed by

$$(w_m)_{PL} = \frac{M_{pc}}{P} - w_i \qquad (3)$$

For a nonlinear M-P-Φ relation, the analytical procedure is essentially the same as for an elastic-perfectly plastic M-P-Φ relation except that the nonlinear M-P-Φ curve has a smooth transition range between elastic and plastic. The internal resistance of the section in this range may be obtained by numerical technique or empirical expressions (2).

5. Numerical Results

Typical numerical results using elastic-perfectly plastic and nonlinear type of M-P-Φ curves are shown in Fig. 2. It can be seen from the figure that the ultimate strengths of beam-column analyzed by using the nonlinear M-P-Φ are significantly lower than the other. This is because the nonlinear part of M-P-Φ curve results in a smooth transition curve near the peak portion of P-w curve. It is plain that the more pronounced the nonlinearity of the M-P-Φ relation, the smoother, and thus the less peak load, the load-deflection curve is likely. Thus, if bi-linear M-P-Φ relation is accepted as a simplification for practical analysis, the ultimate strength of a member may be considerably overestimated.

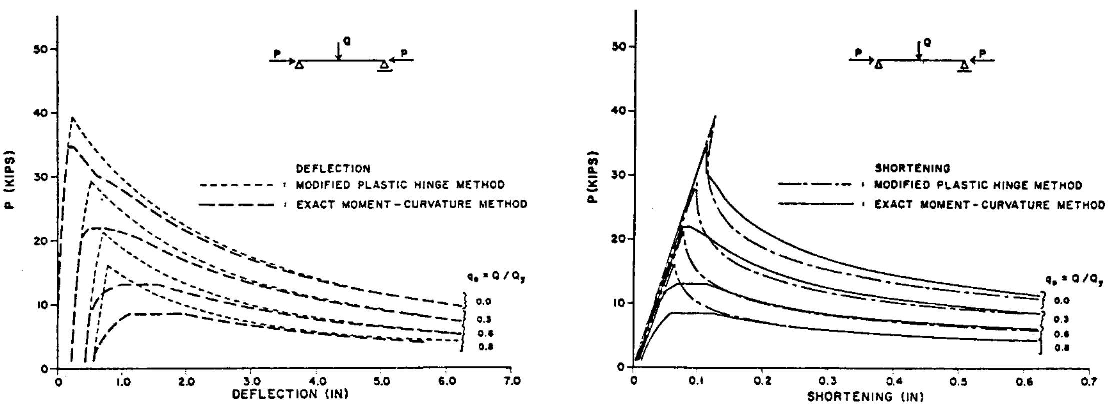

Figure 2 . Comparison between Exact Moment-Curvature Method and Modified Plastic Hinge Method, Pin-Ends, L/r=80

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2. Toma, S., and Chen, W. F., "Inelastic Cyclic Analysis of Pin-ended Tubes," Journal of the Structural Division, ASCE, Vol. 108, No. ST10, Proc. Paper 17423, Oct., 1982, pp. 2279-2294.