CYCLIC ANALYSIS OF FIX-ENDED BEAM-COLUMNS

Hokkai-Gakuen Univ., Member, Shouji Toma
Purdue Univ., Wai F. Chen

The general behavior of a fix-ended column under one cycle of reversed loading can be divided into nine stages: (1) initial elastic loading in compression, (2) one hinge formed in compression, (3) both hinges formed in compression, (4) elastic unloading in compression, (5) elastic loading in tension, (6) one hinge formed in tension, (7) both hinges formed in tension, (8) yielding in tension, and (9) elastic unloading in tension. Figure 1 shows schematically the nine stages of the load-deformation behavior, and the corresponding deformation patterns are shown in Tables 1 and 2. This "hinge-by-hinge" approach accomodates inelastic beam-column problem under cyclic loading to a series of elastic analyses.

The curvature at a cross section is related to bending moment by the relation

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \qquad (1)$$

Solving this equation for a column in compression and tension, the deflections can be expressed in the general forms given in Tables 1 and 2. The integration constants are to be determined by using appropriate boundary conditions and the results are given in the tables.

In tracing the behavior at each stage of loading, the slopes or relative rotations at the ends and at the center will be found by taking derivative of the deflections.

$$\theta_E = \left(\frac{dy_1}{dx}\right)_{x=0} \qquad (2)$$

$$\theta_c = \left(\frac{dy_2}{dx}\right)_{x=L/2} \qquad (3)$$

It should be noted that these slopes or rotations will not change when the column behaves elastically. They are used as the boundary conditions in the subsequent stage.

A typical result of the calculation is shown in Fig. 2. It can be seen from the figure

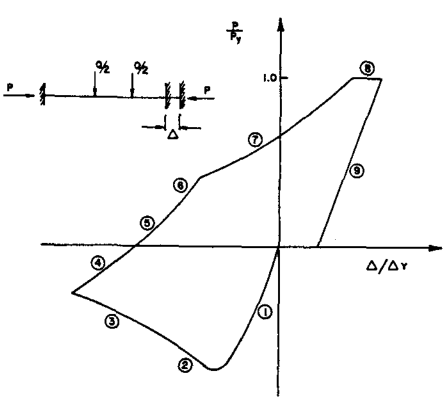


Fig. 1. Stages of Fix-Ended Column Behavior under 1-Cycle of Loading.

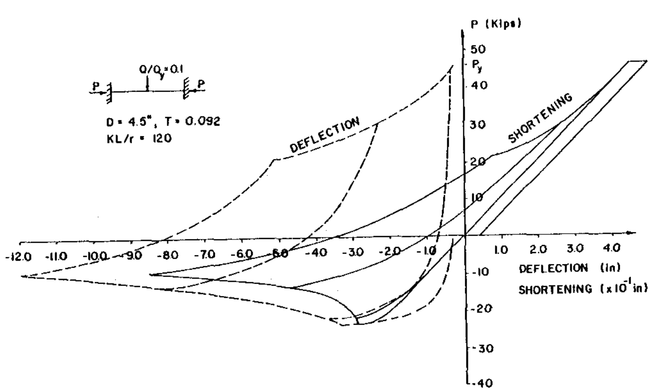


Fig. 2. Cyclic Behavior of Fix-Ended Column, KL/r = 120.

that the post-buckling and plastic tension branches are fixed, while the elastic unloading-tension branch connects these two fixed envelopes. The sooner the axial load is reversed, the closer the slope of the curve is to the elastic slope. The basic concept of the present procedure can be easily extended to the case of more cycles of axial loading, or other type of supporting conditions.

REFERENCE:

Timoshenko, S. P., and Gere, J. M., "Theory of Elastic Stability," Mcgraw-Hill, New York, 1961.

Table 1. Deflections of Each Loading Stage in Compression

$y_1 = A \sin kx + B \cos kx - \frac{Q}{2P} x + \frac{M_E}{P}$ for $x \leq a$, $y_2 = C \sin kx + D \cos kx - \frac{Qa}{2P} + \frac{M_E}{P}$ for $x \geq a$

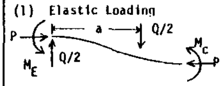
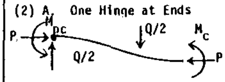
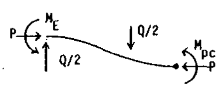
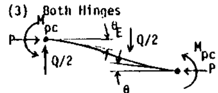
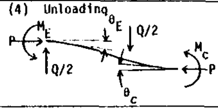
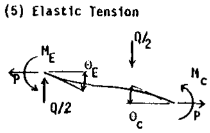
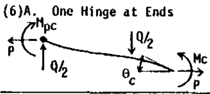
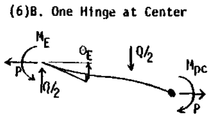
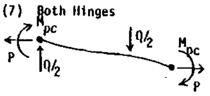
Stages	M_E	A	B	C	D
(1) Elastic Loading 	$\frac{Q}{2k}(\cos ka \cot \frac{kL}{2} + \sin ka - \cot \frac{kL}{2})$	$\frac{Q}{2kP}$	$-\frac{M_E}{P}$	$\frac{Q}{2kP}(1 - \cos ka)$	$\frac{Q}{2kP} \sin ka - \frac{M_E}{P}$
(2) A. One Hinge at Ends 	M_{pc}	$C + \frac{Q}{2kP} \cos ka$	$-\frac{M_{pc}}{P}$	$(\frac{Q}{2kP} \sin ka - \frac{M_{pc}}{P}) \tan \frac{kL}{2}$	$\frac{Q}{2kP} \sin ka - \frac{M_{pc}}{P}$
(2) B. One Hinge at Center 	$- M_{pc} + \frac{Qa}{2} + py_c$	$\frac{Q}{2kP}$	$-\frac{M_E}{P}$	$\frac{Q}{2kP}(1 - \cos ka)$	$\frac{Q}{2kP} \sin ka - \frac{M_E}{P}$
	$y_c = -\frac{M_{pc}}{P}(\frac{1}{\cos\frac{kL}{2}} - 1) + \frac{Q}{2kP}(\tan \frac{kL}{2} - \cos ka \tan \frac{kL}{2} + \sin ka - ka)$				
(3) Both Hinges 	M_{pc}	$\frac{Q}{2kP}(\cos ka - \sin ka \cot \frac{kL}{2}) + \frac{M_{pc}}{P}(\frac{1}{\sin\frac{kL}{2}} + \cot \frac{kL}{2})$	$-\frac{M_{pc}}{P}$	$A - \frac{Q}{2kP} \cos ka$	$\frac{Q}{2kP} \sin ka - \frac{M_{pc}}{P}$
(4) Unloading 	$\frac{P}{k \sin \frac{kL}{2}}[\frac{Q}{2P}(\sin ka \sin \frac{kL}{2} + \cos ka \cos \frac{kL}{2} - \cos \frac{kL}{2}) + \theta_c - \theta_E \cos\frac{kL}{2}]$	$\frac{Q}{2kP} + \frac{\theta_E}{k}$	$-\frac{M_E}{P}$	$\frac{Q}{2kP}(1 - \cos ka) + \frac{\theta_E}{k}$	$\frac{Q}{2kP} \sin ka - \frac{M_E}{P}$

Table 2. Deflections of Each Loading Stage in Tension

$y_1 = E e^{kx} + F e^{-kx} + \frac{Q}{2P} x - \frac{M_E}{P}$ for $x < a$, $y_2 = G e^{ka} + H e^{-ka} + \frac{Qa}{2P} - \frac{M_E}{P}$ for $x > a$

Stages	M_E	E	F	G	H
(5) Elastic Tension 	$\frac{1}{e^{\frac{kL}{2}} - e^{-\frac{kL}{2}}} \left\{ \frac{2P}{k} \theta_c + \frac{Q}{2k}[(1 - e^{ka}) e^{-\frac{kL}{2}} + (1 - e^{-ka}) e^{\frac{kL}{2}}] - \frac{P}{k} \theta_E (e^{\frac{kL}{2}} + e^{-\frac{kL}{2}}) \right\}$	$\frac{M_E}{2P} - \frac{Q}{4kP} + \frac{\theta_E}{2k}$	$\frac{M_E}{2P} + \frac{Q}{4kP} - \frac{\theta_E}{2k}$	$E + \frac{Q}{4kP} e^{-ka}$	$F - \frac{Q}{4kP} e^{ka}$
(6)A. One Hinge at Ends 	$- M_{pc}$	$\frac{1}{e^{\frac{kL}{2}} + e^{-\frac{kL}{2}}} [\frac{\theta_c}{k} - \frac{M_{pc}}{P} e^{-\frac{kL}{2}} - \frac{Q}{4kP} (e^{-ka} e^{\frac{kL}{2}} + e^{ka} e^{-\frac{kL}{2}})]$	$- E - \frac{M_{pc}}{P}$	$E + \frac{Q}{4kP} e^{-ka}$	$F - \frac{Q}{4kP} e^{ka}$
(6)B. One Hinge at Center 	$\frac{1}{e^{\frac{kL}{2}} + e^{-\frac{kL}{2}}} \left\{ 2M_{pc} + \frac{Q}{2k}[(1 - e^{-ka}) e^{\frac{kL}{2}} - (1 - e^{ka}) e^{-\frac{kL}{2}}] - \frac{P}{k} \theta_E (e^{\frac{kL}{2}} - e^{-\frac{kL}{2}}) \right\}$	$\frac{M_E}{2P} - \frac{Q}{4kP} + \frac{\theta_E}{2k}$	$\frac{M_E}{2P} + \frac{Q}{4kP} - \frac{\theta_E}{2k}$	$E + \frac{Q}{4kP} e^{-ka}$	$F - \frac{Q}{4kP} e^{ka}$
(7) Both Hinges 	$- M_{pc}$	$\frac{1}{e^{\frac{kL}{2}} - e^{-\frac{kL}{2}}} [-\frac{M_{pc}}{P} (1 + e^{-\frac{kL}{2}}) + \frac{Q}{4kP} (e^{ka} e^{-\frac{kL}{2}} - e^{-ka} e^{\frac{kL}{2}})]$	$-E - \frac{M_{pc}}{P}$	$E + \frac{Q}{4kP} e^{-ka}$	$F - \frac{Q}{4kP} e^{ka}$