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DISCRETE OPTIMUM DESIGN OF STEEL FRAMED STRUCTURES UNDER MULTIPLE CONSTRAINTS

Keunjoo Byun, Yonsei University
Hakjoo Hwang, Yonsei University

The main objective of this study is to develope a discrete formulation and a solution method for optimum elastic design of framed structures under multiple constraints including member buckling constraints.

1. Basic concept of discrete optimum design.

The objective function and constraints for the binary programming formulation are obtained by using a Taylor series expansion to linearize the equations as follows.

For the (r+1) th design :

$$\begin{aligned} &\text{minimize} && \nabla F(\vec{X}^r)\,\vec{X} \\ &\text{subject to} && G(\vec{X}^r) - J_x \cdot \vec{X}^r + J_x \vec{X} \end{aligned} \qquad (1)$$

where $\vec{X}$: design variables

$F(\vec{X}^r)$: a gradient vector of $F(\vec{X})$

$G(\vec{X})$: design constraint vector

Jx : the Jacobian matrix of G

If there are P available sections for each member, the binary programming formulation for the (r+1) th design is :

$$\begin{aligned} &\text{minimize} && \nabla F(\vec{X}^r)\,\hat{Z}\vec{d} \\ &\text{subject to} && G(\vec{X}^r) - Jx\,\vec{X}^r + Jx\,\hat{Z}\vec{d} \geq \phi \end{aligned} \qquad (2)$$

where $\vec{X} = \hat{Z}\vec{d}$, $U\vec{d} = \vec{e}$, d_{ij} = 0 or 1 (binary variable)

Z_{ik} = Capacity of section j for member i

$\vec{e}$: Unity vector of multiple choice constraints

U : Quasi diagonal matrix having $[1, 1, \; \cdots\cdots]_{1xp}$

2. Formulation of discrete structural design

(1) Objective function.

The cost objective function $F(\vec{d})$ is

$$F(\vec{d}) = \sum_{i=1}^{P} L_i \sum_{k=1}^{P} C_{ik}\, d_{ik} = L^T \hat{c}\, \vec{d} \qquad (3)$$

where L : length vector , $\hat{c}$: Unit cost matrix of available sections

(2) Design Constraints

a. stress constraints

The stress constraints are derived from the AISC inequaity stress (axial + bending) formula and the member buckling theory.

$$G(\vec{X}) = \vec{Y}\vec{X} - \Gamma X\bar{K}A\,(A^T x\,\bar{K}A)^{-1} P' \geq \phi \qquad (4)$$

where $Y = \begin{bmatrix} Y_1 & & \\ & \ddots & \\ & & Y_m \end{bmatrix}$, $Y_i = [1, 1]^T$, $\Gamma = \begin{bmatrix} \Gamma_1 & & \\ & \ddots & \\ & & \Gamma_m \end{bmatrix}$, $\Gamma = \begin{bmatrix} S_i\,\alpha_{Ai}\,E_i\,T_i \\ S_i\,\alpha_{Bi}\,E_i \end{bmatrix}$

P' : a vector of applied joint load in global coordinates

A : branch - node incidence matrix , α_i: sign coefficient

T : translation matrix , K : scaled stiffness matrix

E_i : extractor matrix , $S_i = \frac{x_i}{P_{XA}}$: constant matrix

P_{XA}: allowable axial force of a member

b. Deflection constraints

$$G^a(\vec{X}) = U_u' - (A^T X \overline{K}A)^{-1} p' \geq \phi, \quad G^b(\vec{X}) = -U_e' + (A^T X \overline{K}A)^{-1} p' \geq \phi \qquad (5)$$

U_e', U_u' : a vector of lower and upper limits of joint displacement.

3. Successive binary pogramming

$$\text{minimize } L^T \hat{c}\,\vec{d}, \quad \text{subject to } \left\{ \begin{bmatrix} G(X^r) \\ G^a(J_{xr}) \\ G^b(X^r) \end{bmatrix} - \begin{bmatrix} J_{xr} \\ J_{xr}^a \\ -J_{xr}^a \end{bmatrix} X^r \right\} + \begin{bmatrix} J_{xr} \\ J_{xr}^a \\ -J_{xr}^a \end{bmatrix} \hat{z}\,\vec{d} \geq \phi \qquad (6)$$

$$U\vec{d} = \vec{e}$$

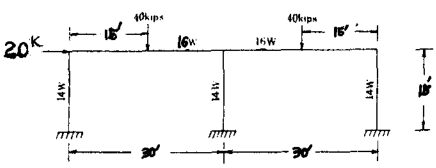
where $J_{xr} = \vec{Y} - \vec{Q}^r C^r$, $\vec{Q}^r = \Gamma\left[I - X^r \overline{K} A (A^T X^r \overline{K} A)^{-1} A^t\right]$

$C^r = \overline{K}A (A^T X^r \overline{K} A)^{-1} p'$, $J_{xr}^a = (A^T X^r \overline{K}A)^{-1} A^T \overline{C^r}$, $J_{xr}^b = -J_{xr}^a$

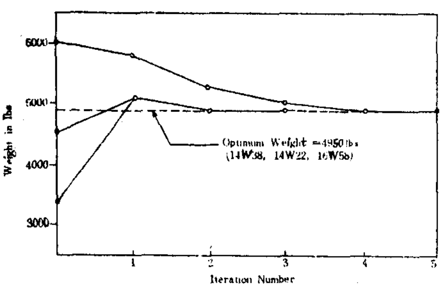
4. Implementation and example problems

The program developed in previous sections, which consists of a main program and four subroutines, solves the discrete optimum design problem for rigid framed systems, that is, planar frames, grids, and space frames.

A one-story, two-bay frame is solved using the program and the results obtained for various problem canditions are discussed in detail.



One-story, two-bay frame



Results with Starting Point Varied

Available Section for One-Story, Two-Bay Frames

Section	Iz in⁴	A in²	Section	Iz in⁴	A in²
14W22	198	6.49	16W26	300	7.67
14W26	244	7.67	16W31	374	9.13
14W30	290	8.83	16W36	447	10.6
14W34	340	10.0	16W40	517	11.8
14W38	386	11.2	16W45	584	13.3
14W43	429	12.6	16W50	657	14.7
14W48	485	14.1	16W58	748	17.1
14W53	542	15.6	16W64	836	18.8
14W61	641	17.9	16W71	941	20.9
14W68	724	20.0	16W78	1050	23.0
14W74	797	21.8	16W88	1220	25.9
14W78	851	22.9	16W96	1360	28.2

Results with Bandwidth and Section Table length

Length	8			12		
Band width	3	5	7	3	5	7
Ietratien	1	3	3	1	3	3
Binary Cycle	28	137	339	28	119	253
Optimum Weight (lb)	4530	4950	4950	4530	4950	4950

In most cases the process converged rapidly to a feasible design, although the results obtained from various starting points show significant diversity. The results indicate that selection of a good starting point will reduce greatly the binary programming time and the total computing time required. If convergence is achieved, it requires few iterations (5 at most for this example).

The use of small bandwidths greatly reduces the binary programming time and the total time. Variation of table length does not affect the time significantly provided that the bandwith about a starting point does not range outside the table.

4. Conclusions

From the results of design axamples using this program, it has been concluded that in the discrete optimum design of steel framed structures the buckling constraints should be included and could be easily incorporated into the general frame work of the formulation, and yet it has the same level of efficiency as the previous research on discrete optimization which is formulated without considering instability.