I-279 SIMULATION OF A SET OF CORRELATED HIGH WIND VELOCITY *

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1. Theory

Simulation of a set of stationary Gaussian processes, x(t) and y(t) with zero means can be attained with prescribed spectral characteristics; individual power spectra, $S_x^c(\omega)$ and $S_y^c(\omega)$ and cross spectrum, $S_{xy}^c(\omega) = C_{xy}^c(\omega) - Q_{xy}^c(\omega)$ by the following mathematical models.

$$x(t) = \sum_{j=-N}^{N} a_j e^{i(\omega_j t + \phi_j)} \quad \ldots\ldots (1) \qquad \text{and } y(t) = \sum_{j=-N}^{N} b_j e^{i(\omega_j t + \eta_j)} \quad \ldots\ldots\ldots (2)$$

where the following conditions are made;

(1) a_j; complex variables, $a_j = A_j e^{i\theta_j}$ and $A_j = \sqrt{S_x^c(\omega_j)\Delta\omega}$, $\theta_j = \tan^{-1}\left\{ -\dfrac{Q_{xy}^c(\omega_j)}{C_{xy}^c(\omega_j)} \right\}$ ...(3)

$\underline{a}_j = a_j^*$, a_j^* is the complex conjugate of a_j. $a_0 = 0$

(2) b_j; real variables, $b_j = \sqrt{S_y^c(\omega_j)\Delta\omega}$ ...... (4), $b_{-j} = b_j$, $b_j = 0$

$\omega_j = \omega_L + (j - 1/2)\Delta\omega$, $\Delta\omega = (\omega_U - \omega_L)/N$ .... (5) $\omega_{-j} = -\omega_j$

ω_L and ω_U are lower and upper wave numbers(circular frequencies) cut off for the significant spectral intensity. N is a large number.

(3) ϕ_j and η_j; random variables with uniform distribution over the range(0, 2π). ϕ_j and η_j are correlated and the joint probability density function is given by

$$p(\phi_j , \eta_j) = \frac{1}{4\pi^2}\left\{1 + \frac{a_{xyj}}{\pi^2}(\phi_j - \pi)(\eta_j - \pi)\right\} \text{ where } a_{xyj} = \pi^2 \frac{|S_{xy}^c(\omega_j)|}{\sqrt{S_x^c(\omega_j)}\sqrt{S_y^c(\omega_j)}} \quad ..(6)$$

$\phi_{-j} = -\phi_j$, $\eta_{-j} = -\eta_j$

With the conditions above described, eqs(1) and (2) can be put in the forms:

$$x(t) = 2\sum_{j=1}^{N} A_j \cos(\omega_j t + \theta_j + \phi_j) \quad \ldots. (7) , \quad y(t) = 2\sum_{j=1}^{N} b_j \cos(\omega_j t + \eta_j) \quad \ldots. (8)$$

For the simulation, eqs(7) and (8) can be used.

<u>Proof;</u> It is easy to show that eqs(1) and (2) are stationary Gaussian processes with zero means. Consider the cross correlation function of x(t) and y(t). For the complex functions, it will be given by

$$E[x(t+\tau)y^*(t)] = \sum_{j=-N}^{N} \frac{a_j b_j a_{xyj}}{\pi^2} e^{i\omega_j \tau} = R_{xy}(\tau) \quad \ldots.. (9)$$

Thus, the cross spectral density function is with eq(9),

$$S_{xy}(\omega) = 1/2\pi \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-i\omega\tau} d\tau = \sum_{j=-N}^{N} a_j b_j a_{xyj} / \pi^2 \, \delta(\omega - \omega_j)$$

* Taken from ' Two Stochastic Models for Simulation of Correlated Random Processes', M.Hoshiya and H.W.Tieleman, VPI-E-71-9 Tech. Report, June 1971, NASA Grant No. NGL 47-004-067. ** Assistant Professor, Ph.D. *** Assistant Professor, Ph.D.

Substitution of eqs(3) to(6) in the above expression gives

$$S_{xy}(\omega) = \sum_{j=-N}^{N} |S^c_{xy}(\omega_j)| \Delta\omega \, e^{i\theta_j} \, \delta(\omega - \omega_j)$$

Thus, taking the limit, $\lim_{N\to\infty} S_{xy}(\omega) = \int |S^c_{xy}(\omega_j)| \, e^{i\theta_j} \, \delta(\omega - \omega_j) \, d\omega_j = |S^c_{xy}(\omega)| \, e^{i\theta_j} = S^c_{xy}(\omega)$.

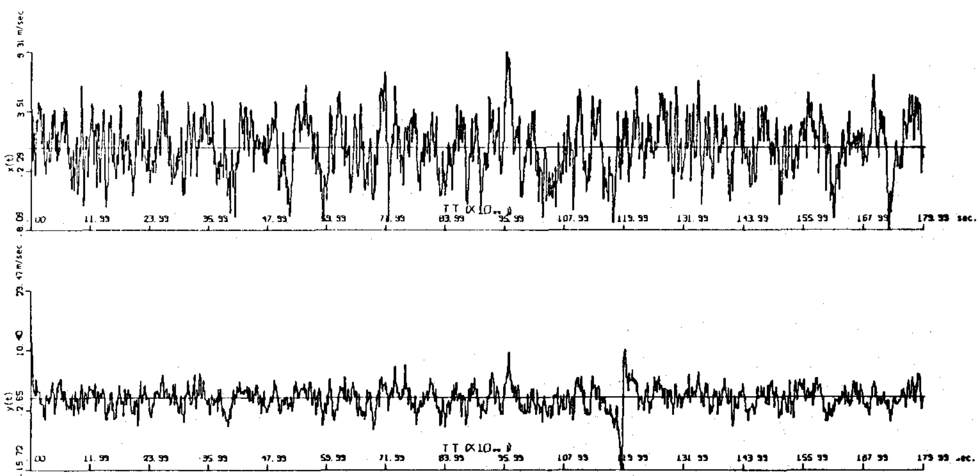
Therefore, when N is large, the crossspectral density function of eqs(1) and (2) becomes the prescribed spectrum $S^c_{xy}(\omega)$.

Simulation Procedure; The following procedure will be used to generate sample histories of x(t) and y(t). Given are the power spectral density functions of each of the two random processes $S^c_x(\omega)$ and $S^c_y(\omega)$ and their cross spectral density function $S^c_{xy}(\omega)$. The significant part of these spectra between a lower frequency ω_L and an upper frequency ω_U is subdivided in N intervals of width $\Delta\omega$. The prescribed spectra are evaluated at the center of each interval such that $\omega_j = \omega_L + (j - 1/2)\Delta\omega$. Deterministic values for A_j, b_j, a_{xyj} and θ_j can be obtained for $j = 1,2,\dots,N$. The random variables ϕ_j and η_j are generated from the joint probability density function $p(\phi_j, \eta_j)$. We can now proceed to calculate a point of the random processes, say at time t_1. Next, the following point of the random processes can be calculated at time $t_1 + \Delta t$ and so on. The choice of the time interval depends on the value of the upper significant wave number and should be chosen such that $\Delta t \leq \pi/\omega_U$.

2. Streamwise Strong Wind Turbulence Component

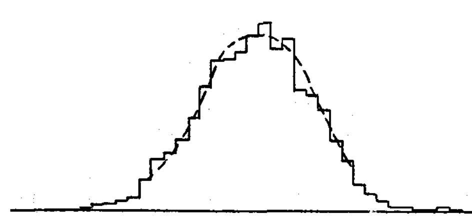
Let the flow field be homogeneous in the horizontal plane and stationary with respect to time. The wind velocity can be resolved into three components, two in the horizontal plane, one of which is along the mean wind direction(x) and one is normal to it(y). The third component is taken vertically upward(z). The instantaneous components can be written in the forms of a mean and a fluctuation, so that $U(z,t) = \bar{U}(z) + u(z,t)$, $V(z,t) = v(z,t)$, $W(z,t) = \bar{W}(z) + w(z,t)$. The estimated spectral density function of the wind speed in the planetary boundary layer for the eastern United States is given by van der Hover. It shows that the spectral gap between 1 cycle per hour to 10 cycles per hour may be used for the stable estimation of $\bar{U}$ and $\bar{W}$ for a sample function with a duration of about 1/2 hour. Under strong wind conditions the mixing action of the mechanical turbulence tends to reduce the atmosphere to a state of neutral stability. Consequently the turbulence near the ground is only due to frictional effects and will vary only significantly with the surface drag and height above the ground. If this is the case, the turbulence near the ground can be described as a random process which may be approximated by a stationary Gaussian process with zero mean. The correlated longitudinal turbulence-components at two different locations A and B are simulated. The direction of the separation distance AB is normal to the x coordinate in the horizontal plane and consequently parallel to the y coordinate.

For the prescribed spectrum, Davenport's empirical strong wind spectrum is used. After modifining it to the theoretical spectrum, $S_u(\omega) = 1/2\ S_u'(\omega) = \left(2k\bar{U}_1^2 / \omega\right)\left(x^2 / (1+x^2)^{4/3}\right)$ where $x = 600\omega/\pi\bar{U}_1$ and k is the drag coefficient. $\bar{U}_1$ is the reference mean-velocity near the ground at an elevation of 10 meters. The coherence function for the longitudinal turbulence components at two points separated by a distance Δy parallel to the y coordinate is from Ang and Amin, $\gamma(\omega) = \exp\left\{- k'\Delta y\omega/ 2\pi\bar{U}_1\right\}$ where k' is a dimensionless coefficient between 20 and 25. For this particular case, the maximum correlation between u_A and u_B is expected to be occurred at a zero lag and consequently it is assumed that the quadrature spectrum function, $Q_{xy}(\omega)$ is zero for all ω. The simulation is carried out and the set of sample records is shown below.

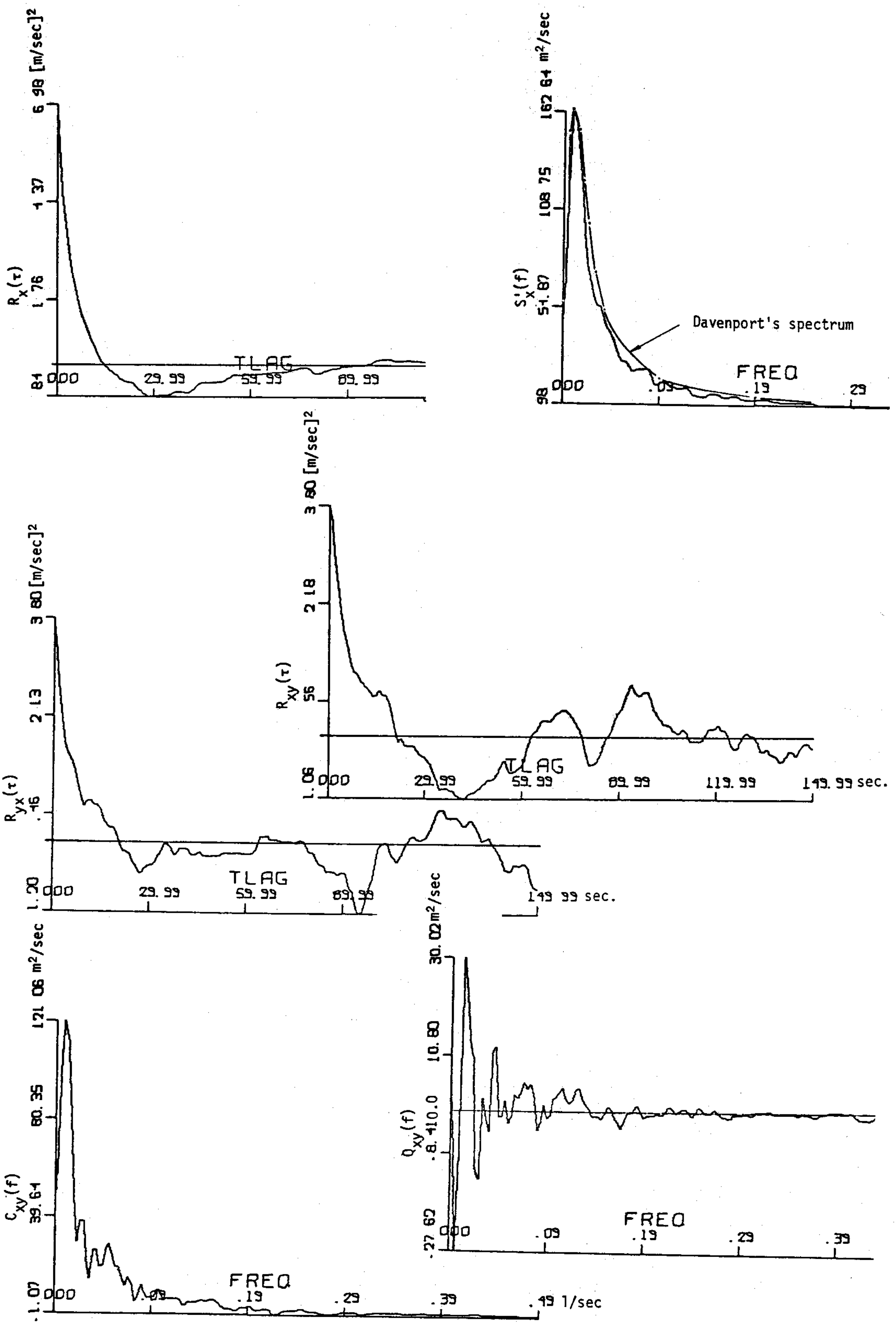


The number of generated random variables is N = 600. Davenport's spectrum was evaluated at intervals of $\Delta\omega$ = .005236 rad/sec with a lower wave number cut-off ω_L = 0.00377 rad/sec(= 0.00 06 Hz) and an upper wave number cut-off ω_U = 3.14 rad/sec(= 0.5Hz). All simulated turbulence data were evaluated at Δt = 1.0 sec time interval in order to avoid the aliasing. The total length of each of the simulated sample records is thirty minutes. The average reference velocity at altitude of 10 meters was taken to be $\bar{U}_1$ = 16.5 m/sec. The drag coefficient k was chosen to be 0.005 and the lateral separation distance Δy was taken at 5 meters.

The right figure shows the hystogram of the amplitude of the simulated sample records and is approximately Gaussian in nature. The autocorrelation function, cross correlation functions, power spectrum and cross spectral density function of the simulated records are analyzed and shown.



$\bar{x}$ = 0.025925 m/sec
σ_x = 2.6438 m/sec



6.98 [m/sec]²
$R_x(\tau)$
TLAG
0.00
29.99
59.99
89.99
162.64 m²/sec
108.75
54.87
$S'_x(f)$
Davenport's spectrum
FREQ
.09
.19
.29
3.80 [m/sec]²
2.18
$R_{xy}(\tau)$
TLAG
119.99
149.99 sec.
$R_{yx}(\tau)$
2.13
121.06 m²/sec
80.35
39.64
$C_{xy}(f)$
.39
.49 1/sec
30.02 m²/sec
10.80
$Q_{xy}(f)$
-27.62