I-204 RELIABILITY OF REDUNDANT SYSTEMS

VIRGINIA POLYTECHNIC INSTITUTE MEMBER OF JSCE MASARU HOSHIYA

1. INTRODUCTION For redundant structural systems, particular component failure does not necessarily mean the overall collapse of the systems. The counting of all possible failure paths to obtain the reliability remains a very complicated procedure. A formulation of systematic counting procedure by Yao and Yeh [1] is found to be still of no practical use except for the rather simple systems with low redundancy. This note shows that use of a Monte Carlo method reduces the extreme difficulty of the reliability analysis by sacrificing some degree of accuracy. The Result for a redundant cable system indicates that even the reliability of redundant systems should also be based on the probability of non-component failure.

2. PARALLEL CABLE SYSTEM Consider a structural system of initial m components - a parallel cable system with single random load S. (Fig. 1). We consider herein the case where the material of components is brittle and the components in the structure fail through fracture. Thus, the failed components become completely inactive. As in [1], the component resistances are independent, but identically distributed random variables and share equally the external load S. The Monte Carlo method we employed is considered an experimental study of evaluation of reliability by computer. For the given probability distribution function of the load S and the resistances R_i, i=1,2, · · · ,m of each component, we generate S and R_i, i=1,2, · · ·,m. This technique of random number generation is well known and described elsewhere [2]. For the set of realization, S and R_i,i=1,2,· · ·,m, we examine if the equally distributed member force, S/m exceeds the resistances R_i for i=1,2,· · ·,m. If k components fail, the load S is to be redistributed equally over the remaining components. Thus, each component is now subject to the member force, S/(m-k). Comparison between the remaining resistances and the redistributed member forces is then made repeatedly until the end of the failure paths. If we repeat the above experiment many times, keeping the tally of every possible path, the probability of failure of total i members can be obtained by n_i/N, where n_i is the total numbers of failure of i members and N is the total numbers of repeated experiments. The systematic counting procedure is simple and the flow chart of the Monte Carlo method is illustrated in (Fig. 2). As the numerical example, lognormal distributions of both the external load S and the resistances, R_i are employed. Input data and the results based on 2000 trials for systems with two components to fourteen components are given in (Table 1). Theoretical upper and lower bounds are given in (Table 2). The accuracy and convergency of Monte Carlo results are also shown in (Fig. 3). From these examples, a very important observation can be made; the fact that for instance, the probabilities of failure of five components to thirteen components were zero for the originally fourteen component system indicates the obvious

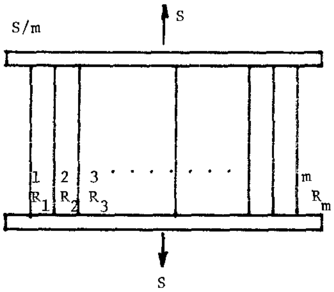

Fig. 1. Redundant Brittle System

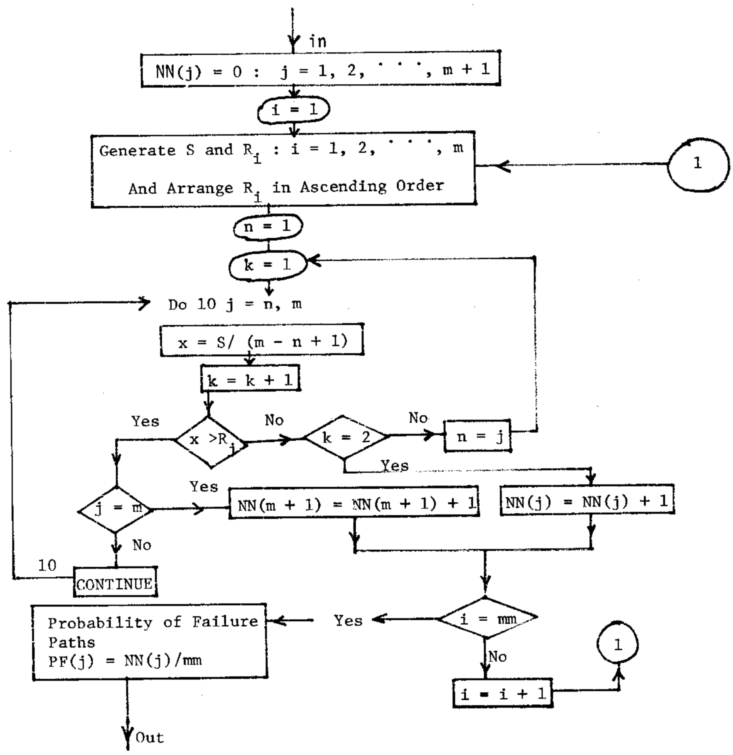

Fig. 2. FLOW CHART OF MONTE CARLO METHOD

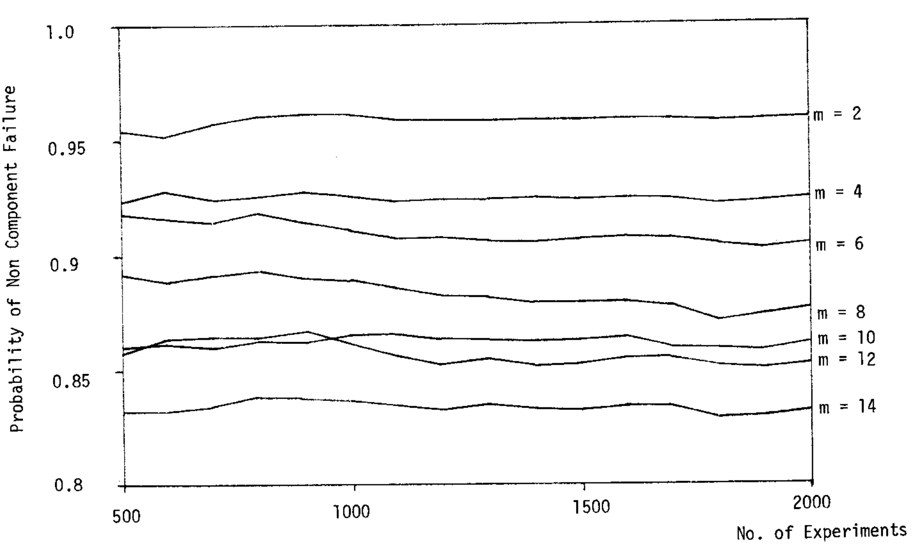

Fig. 3. PROBABILITY OF NON COMPONENT FAILURE VS. NO. OF EXPERIMENTS

chain failure occurrence. In other words, at least five component failure produces a sequency of failure paths all the way until the complete collapse of the system. The same observation is made for the other systems. This fact may suggest that the evaluation of reliability of redundant systems should also be based on the probability of non-component failure. Reliability of non-component failure and Reliability of at least one component success are shown respectively in the last two rows of (Table 2).

Table 1 FAILURE PATHS OF REDUNDANT CABLE SYSTEM (2000 Trials)

Original System m		2	4	6	8	10	12	14
External Load (Kips) LN (,)		(40,10)	(80,20)	(120,30)	(160,40)	(200,50)	(240,60)	(280,70)
Resistance of each Cable (Kips)		LN(40,10)						
Original Member Force		S´ = S/m = LN (20,5)						
Probability of Member Failure	0	0.9580	0.9225	0.9040	0.8760	0.8620	0.8510	0.8295
	1	0.0025	0.0210	0.0285	0.0375	0.0493	0.0545	0.0615
	2	0.0395	0.0005	0.0050	0.0060	0.0087	0.0125	0.0165
	3		0.0 *	0.0 *	0.0 *	0.0020	0.0050	0.0045
	4		0.0560	0.0	0.0	0.0007	0.0010	0.0015
	5			0.0	0.0	0.0 *	0.0 *	0.0 *
	6			0.0625	0.0	0.0	0.0	0.0
	7				0.0	0.0	0.0	0.0
	8				0.0805	0.0	0.0	0.0
	9					0.0	0.0	0.0
	10					0.0773	0.0	0.0
	11	*Chain failure occurrence					0.0	0.0
	12						0.0760	0.0
	13							0.0
	14							0.0865
Reliability based on Non component Failure		0.9580	0.9225	0.9040	0.8760	0.8620	0.8510	0.8295
Reliability based on at least one component success		0.9605	0.9440	0.9375	0.9195	0.9227	0.9240	0.9135

Original System m	2	4	6	8	10	12	14
Upper bound, P'_{ss}	0.9693			same			
Reliability based on Non component Failure P_{ss}	0.9580	0.9225	0.9040	0.8760	0.8620	0.8510	0.8295
Lower bound, P^*_{ss}	0.9386	0.8772	0.8158	0.7544	0.6930	0.6316	0.5702

Table 2 UPPER AND LOWER BOUNDS FOR RELIABILITY OF Non component Failure

3. CONCLUSION Monte Carlo Simulation has a great advantage over the analytical method for the evaluation of probability of failure paths of redundant systems since it is simple and also shows a clear physical interpretation of the failure phenomena. Due to this research, it is found that for a redundant system, obvious chain failure occurs and the system reliability can be given by the non failure probability of any component, which can be easily evaluated even theoretically. This study was done in Engineering Mechanics Department of Virginia Polytechnic Institute and State University.

REFERENCES

[1] Yao, J. T. P. and Yeh, H. Y., Formulation of Structural Reliability, J. of ST12, ASCE, Dec., 1969.

[2] IBM. SSP. Gauss, Randu.

[3] Freudenthal, A. M., Critical Appraisal of Safety Criteria and Their Basic Concepts, Proc. of 8th Conf. IABSE, New York, 1968.

[4] Ang, A. H.-S., and Amin, M., Safety Factors and Probability in Structural Design, J. of ST7, ASCE, July,1969

[5] Ang, A. H.-S., On Concepts of Safety of Structures and Methods of Design, Proc. of IABSE, London, 1969.

[6] Ang, A. H.-S., Extended Reliability Basis of Structural Design Under Uncertainties, 9th Reliability and and Maintainability Conf., Detroit, July, 1970.

[7] Benjamin, J. R., Probabilistic Models for Seismic Force Design, J. of ST5, ASCE, May, 1968.

[8] Benjamin, J. R., Probabilistic Structural Analysis and Design, J. of ST7, ASCE, July, 1968.

[9] Benjamin, J. R., and Cornell, C. A., Probability, Statistics and Decision for Civil Engineers, McGraw-Hill, 1970.

[10] Freudenthal, A. M., Garrelts, J. M., and Shinozuka, M., The Analysis of Structural Safety, J. of ST1, ASCE, Feb., 1966.

[11] Ang, A. H.-S. and Amin, M., Reliability of Structures and Structural Systems, J. of EM2, ASCE, April, 1968.

[12] Heller, R. A., Reliability Through Redundancy? Proc. of 15th Annual Tech. Meeting, Inst. of Environmental Sciences, April, 1969.

[13] Turkstra, C. J., Choice of Failure Probabilities, J. of ST6, ASCE, Dec., 1967.

[14] Costello, J. F. and Chu, K. H., Failure Probabilities of Reinforced Concrete Beams, J. of ST10, ASCE, Oct., 1969.

[15] Jorgenson, J. L., and Goldberg, J. E., Probability of Plastic Collapse Failure, J. of ST8, ASCE, Aug. 1969.

[16] Marshall, P. W., Risk Evaluations for Offshore Structures, J. of ST12, ASCE, Dec., 1969.

[17] Spence, S. T. and Hoshiya, M., Reliability of a Single Flexible Column with Three Spring Supports, J. of JSCE, No. 183, Nov., 1970.

[18] Hoshiya, M. and Spence, S. T., Reliability Analysis of Tainter Gate, J. of JSCE, No. 183, Nov., 1970.

[19] Cornell, C. A., Bounds of the Reliability of Structural Systems, J. of ST1, ASCE, Feb., 1967.

[20] Moses, F. and Kinser, D.E., Optimum Structural Design with Failure Probability Constraints, J. of AIAA, June, 1967.